Gov 50: 14. Regression and Causality (II)

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Fall 2018

1. Today's agenda

2. Heterogeneous treatment effects

3. Non-linear relationships

4. Causality and regression wrap up

1/ Today's agenda

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  - Learning about how to use regression to predict and estimate causal effects.
- Today:
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- HW3 due tonight.

2/ Heterogeneous treatment effects

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##     20.0    41.0    50.0    49.8    59.0   106.0
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```
##    Min. 1st Qu.  Median    Mean 3rd Qu.    Max.
##    20.0    41.0    50.0    49.8    59.0   106.0
```

```
social.neighbors <- subset(social,
                           neighbors == 1 | control == 1)
```

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$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 \text{age}_i + \widehat{\beta}_2 \text{neighbors}_i + \widehat{\beta}_3 (\text{age}_i \times \text{neighbors}_i)$$

Predicted values from non-interacted model

- Let $X_i = \text{age}_i$ and $Z_i = \text{neighbors}_i$:

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 X_i + \widehat{\beta}_2 Z_i$$

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	Control ($Z_i = 0$)	Neighbors ($Z_i = 1$)
25 year-old ($X_i = 25$)		
26 year-old($X_i = 26$)		

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	Control ($Z_i = 0$)	Neighbors ($Z_i = 1$)
25 year-old ($X_i = 25$)	$\widehat{\alpha} + \widehat{\beta}_1 \cdot 25 + \widehat{\beta}_2 0$	
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- Effect of Neighbors for a 25 year-old:

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- Effect of Neighbors for a 25 year-old:

$$(\widehat{\alpha} + \widehat{\beta}_1 25 + \widehat{\beta}_2) - (\widehat{\alpha} + \widehat{\beta}_1 25)$$

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- Effect of Neighbors for a 25 year-old:

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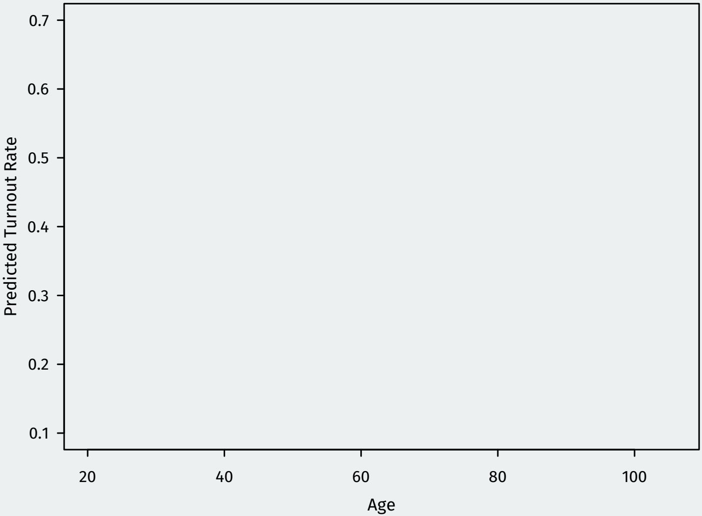
Visualizing the regression



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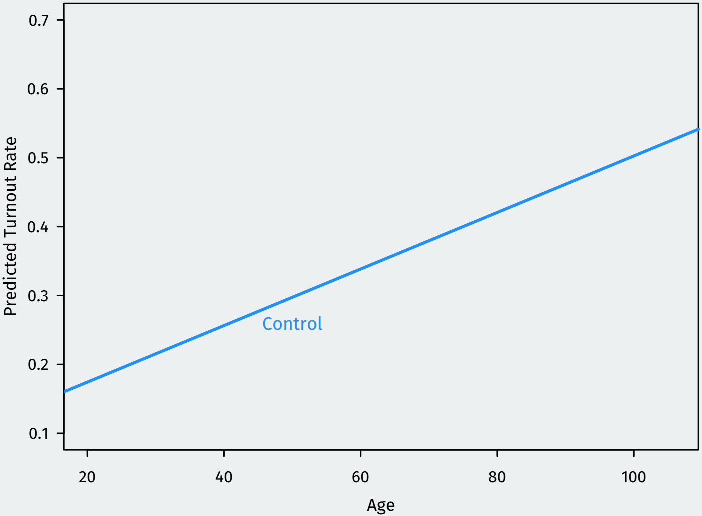

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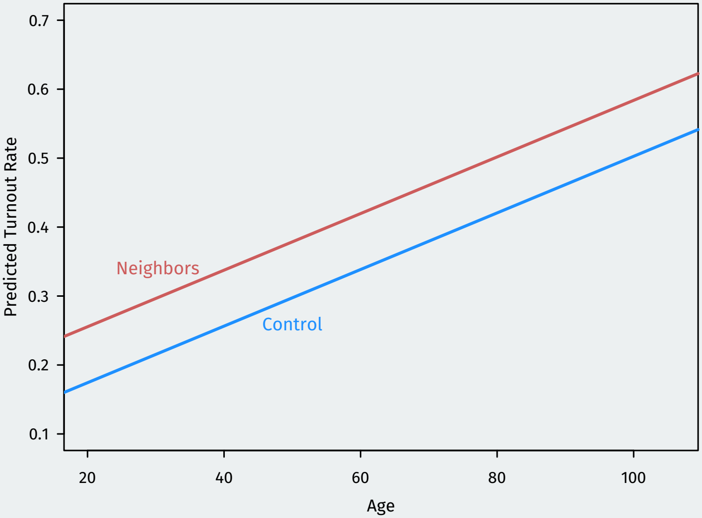

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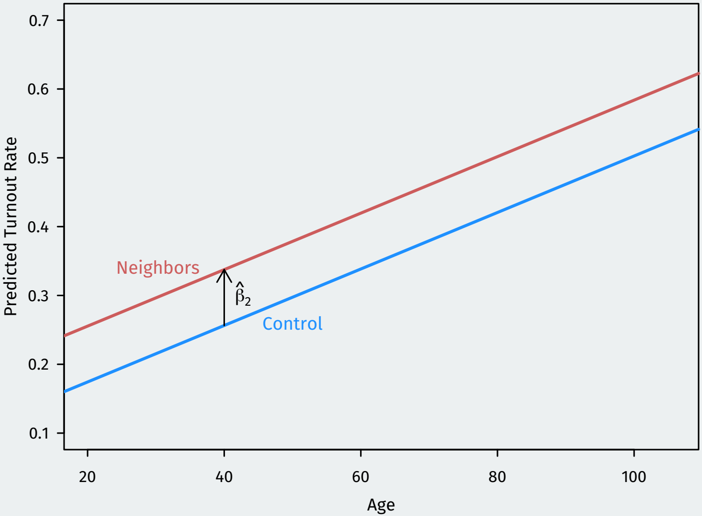

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- Effect of Neighbors for a 25 year-old:

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- Effect of Neighbors for a 25 year-old:

$$(\widehat{\alpha} + \widehat{\beta}_1 25 + \widehat{\beta}_2 + \widehat{\beta}_3 \cdot 25) - (\widehat{\alpha} + \widehat{\beta}_1 25)$$

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- Effect of Neighbors for a 26 year-old:

$$(\widehat{\alpha} + \widehat{\beta}_1 26 + \widehat{\beta}_2 + \widehat{\beta}_3 \cdot 26) - (\widehat{\alpha} + \widehat{\beta}_1 26)$$

Predicted values from interacted model

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 X_i + \widehat{\beta}_2 Z_i + \widehat{\beta}_3 X_i Z_i$$

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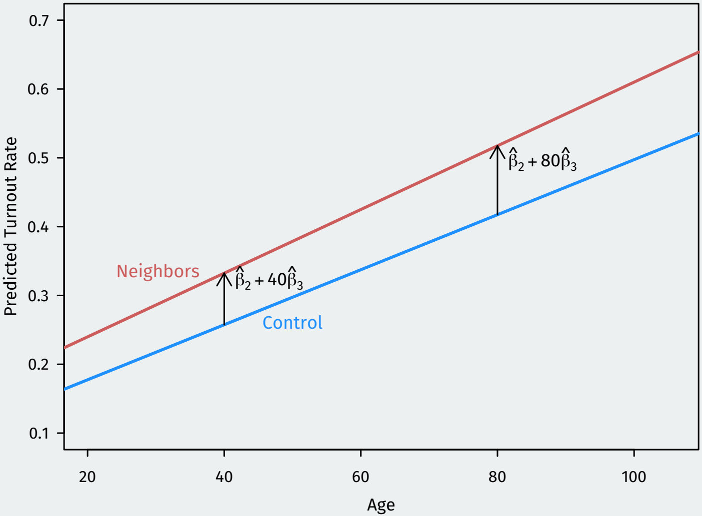
- Effect of Neighbors for a 26 year-old:

$$(\widehat{\alpha} + \widehat{\beta}_1 26 + \widehat{\beta}_2 + \widehat{\beta}_3 \cdot 26) - (\widehat{\alpha} + \widehat{\beta}_1 26) = \widehat{\beta}_2 + \widehat{\beta}_3 \cdot 26$$

- Effect of Neighbors for a x year-old: $\widehat{\beta}_2 + \widehat{\beta}_3 \cdot x$

Visualizing the interaction



Interpreting coefficients

$$\widehat{Y}_i = \widehat{\alpha} + \widehat{\beta}_1 \text{age}_i + \widehat{\beta}_2 \text{neighbors}_i + \widehat{\beta}_3 (\text{age}_i \times \text{neighbors}_i)$$

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- These hold no matter what types of variables they are!

3/ Non-linear relationships

Linear regression are linear

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- Standard linear regression can only pick up **linear** relationships.
- What if the relationship between X_i and Y_i is non-linear?

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- $\widehat{\beta}_2$: how the effect of age increases as age increases.

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age.vals <- 20:85
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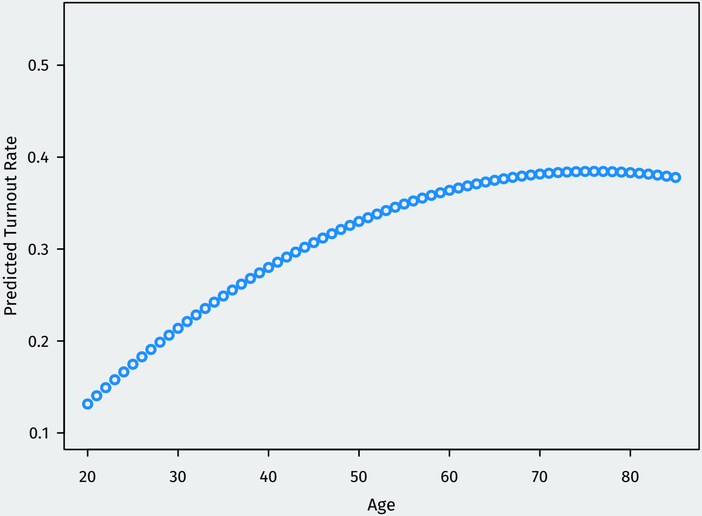
- Plot the predictions:

```
plot(x = age.vals, y = age.preds, ylim = c(0.1, 0.55),
     xlab = "Age", ylab = "Predicted Turnout Rate",
     col = "dodgerblue", lwd = 2)
```

Plotting predicted values



Plotting lines instead of points

- If you want to connect the dots in your scatterplot, you can use the `type = "l"` ("line" type):

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plot(x = age.vals, y = age.preds, ylim = c(0.1, 0.55),
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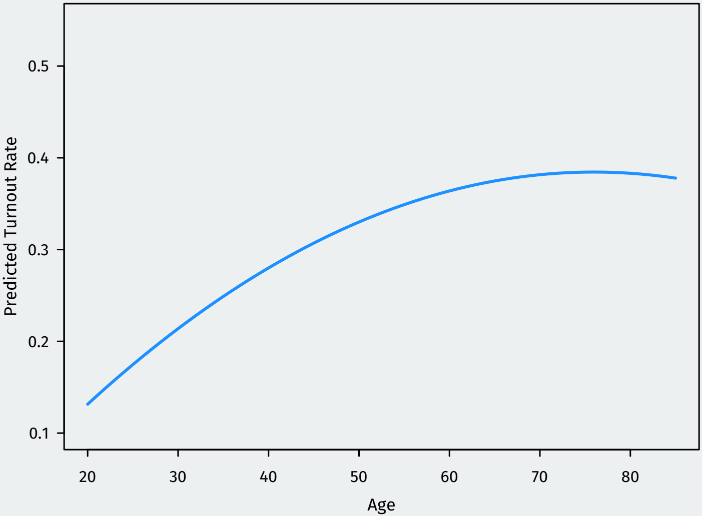
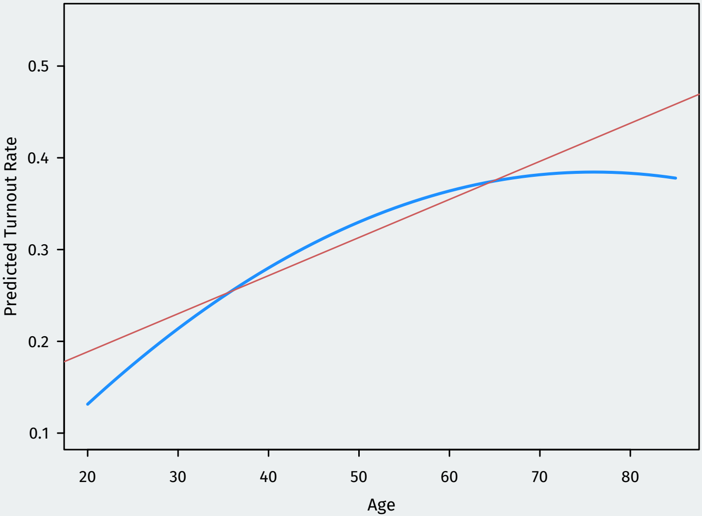


Comparing to linear fit



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```
health <- read.csv("data/health2017.csv")
w.fit <- lm(weight ~ steps.lag + dayofyear, data = health)
```

Residual plot

```
plot(health$steps.lag, residuals(w.fit),
     xlab = "Lagged Steps", ylab = "Residuals")
plot(health$dayofyear, residuals(w.fit),
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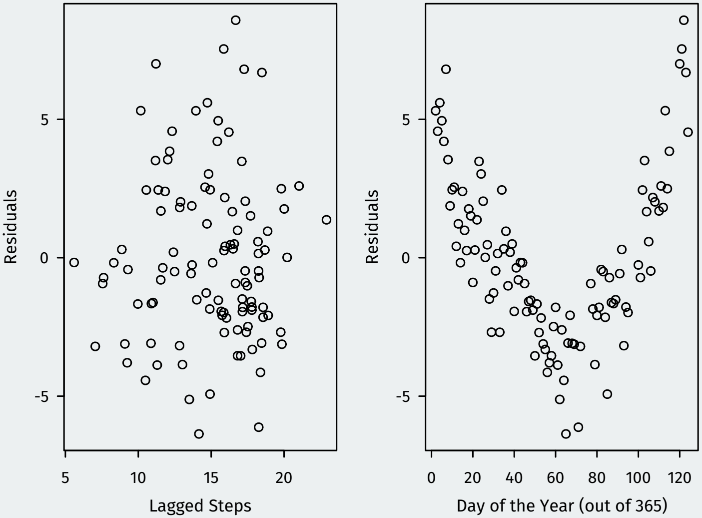
Residual plot



Add a squared term for a better fit

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w.fit.sq <- lm(weight ~ steps.lag + dayofyear + I(dayofyear^2),
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coef(w.fit.sq)
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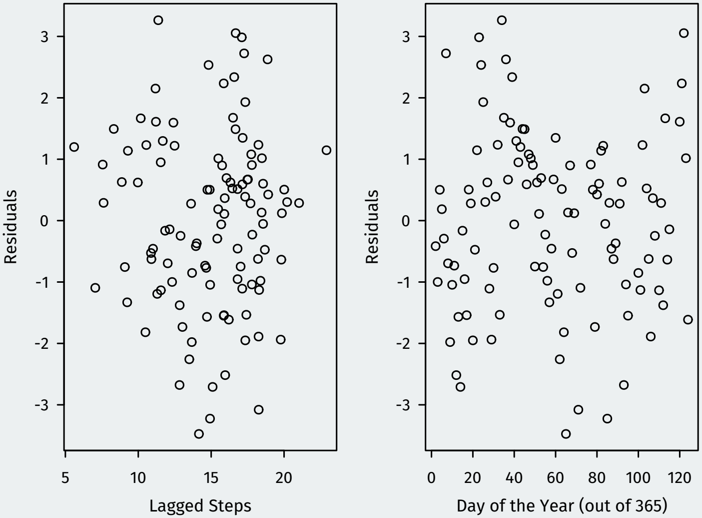
Residual plot, redux



4/ Causality and regression wrap up

Regression and causality

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  - Confounders: other variables that cause both treatment and outcome.
  - Before/after and diff-in-diff designs can be implemented with regression, too.

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- Next part of class: quantifying uncertainty.
  - First stop: probability, the mathematical language of uncertainty.