Gov 50: 18. Estimation: Surveys

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Fall 2018

1/ Today's agenda

Logistics

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  - Practice midterm out now.

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- Lessons today applicable to most statistical procedures.

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  - Approve (40%), Disapprove (54%)
- What can we learn about Trump approval in the population from this one sample?

2/ Samples and estimators

Samples from the population

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  - $\rightsquigarrow$ i.i.d. random variables
  - e.g.: $Y_i = 1$ if i approves of Trump, $Y_i = 0$ otherwise.
- **Statistical inference** is using data to guess something about the population distribution of Y_i.

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- These are the things we want to learn about.

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  - $\leadsto$ $\hat{\theta}$ has a distribution.

3/ Properties of estimators

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- How good are these different estimators?

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- $\theta = p$ and $\hat{\theta} = \overline{Y}$

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- $\rightsquigarrow \overline{Y}$ has a distribution across repeated samples.

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- **Unbiasedness**: Sample proportion is on average equal to the population proportion.

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4/ Confidence intervals

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- **Confidence interval**: way to construct an interval that will contain the true value in some fixed proportion of repeated samples.

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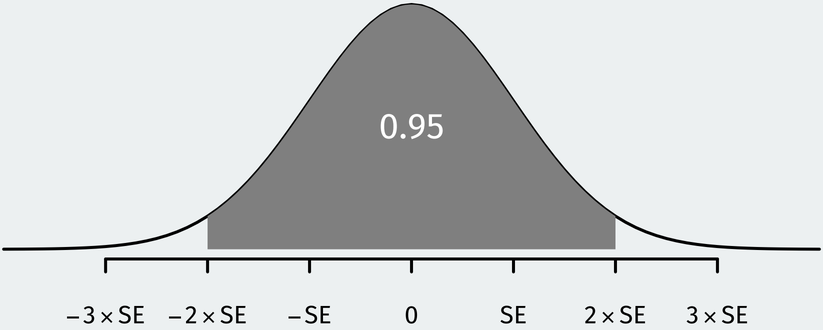
- Chance error: $\overline{Y} - p$ is approximately normal with mean 0 and SE equal to $\sqrt{\frac{p(1-p)}{n}}$

Chance errors



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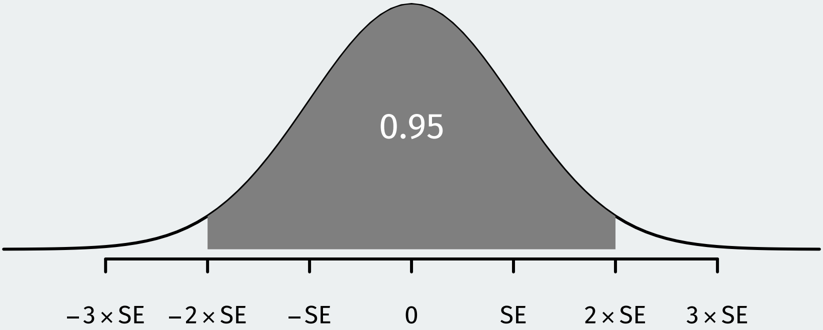

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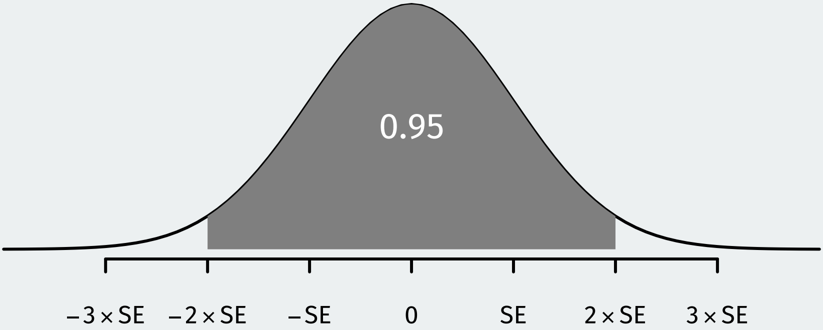

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- $\rightsquigarrow$ range of plausible chance errors is $\pm 1.96 \times SE$

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  - What percent of chance errors do you want to count as "plausible"?
  - Convention is 95%.
- $100 \times (1 - \alpha)\%$ confidence interval:

$$CI = \overline{Y} \pm z_{\alpha/2} \times SE$$

  - In polling, $\pm z_{\alpha/2} \times SE$ is called the **margin of error**
- $z_{\alpha/2}$ is the $N(0, 1)$ z-score that would put $\alpha/2$ of the probability density above it.
  - $\mathbb{P}(-z_{\alpha/2} < Z < z_{\alpha/2}) = \alpha$
  - 90% CI $\rightsquigarrow \alpha = 0.1 \rightsquigarrow z_{\alpha/2} = 1.64$
  - 95% CI $\rightsquigarrow \alpha = 0.05 \rightsquigarrow z_{\alpha/2} = 1.96$
  - 99% CI $\rightsquigarrow \alpha = 0.01 \rightsquigarrow z_{\alpha/2} = 2.58$

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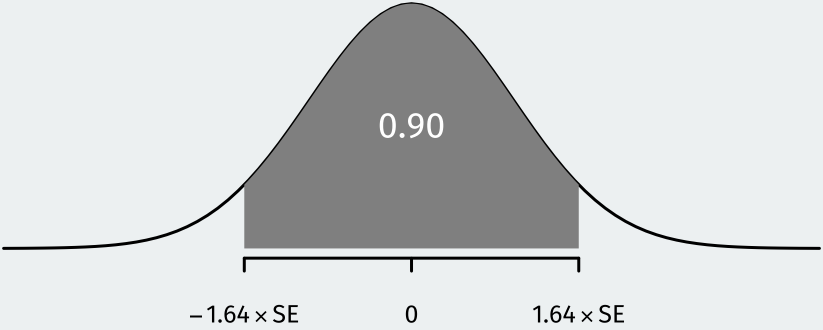
Z-values



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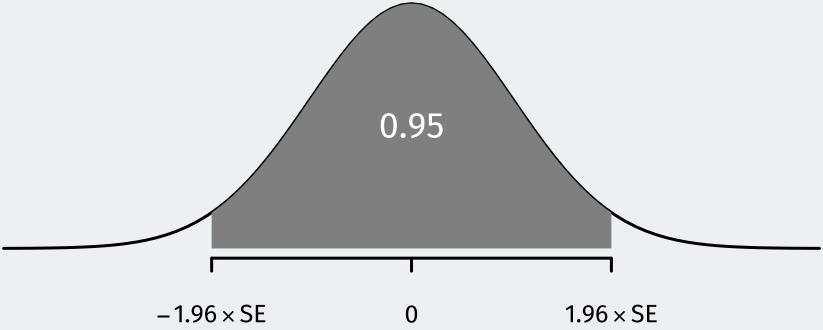

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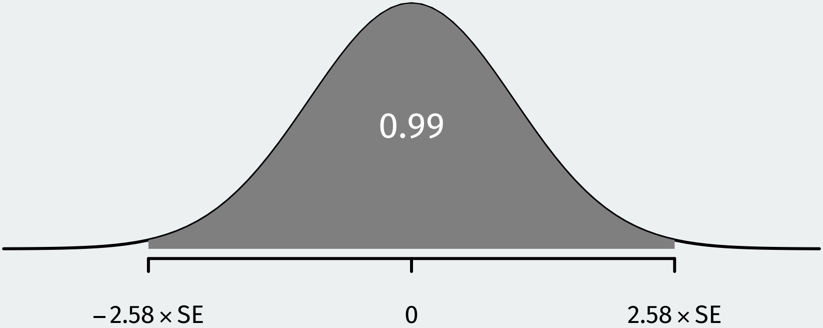

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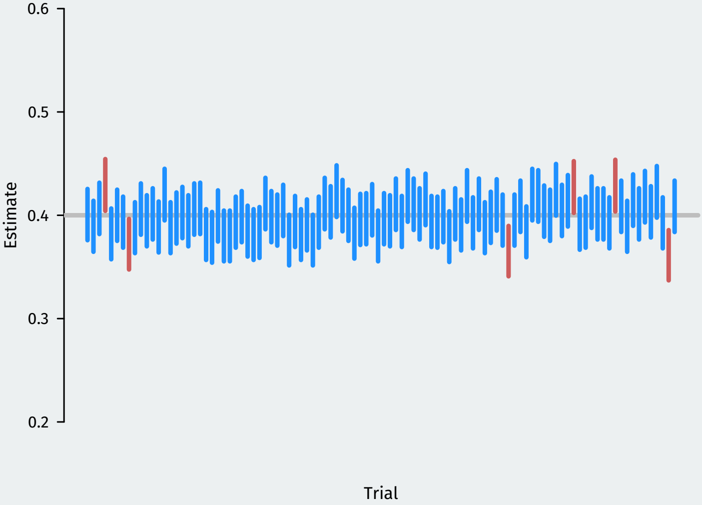

5/ How big of a sample do I need?

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> Gallup tracks daily the percentage of Americans who approve or disapprove of the job Donald Trump is doing as president. Daily results are based on telephone interviews with approximately 1,500 national adults; Margin of error is ±3 percentage points.

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- But we don't know p! $\rightsquigarrow$ use $p = 0.5$ since this require the biggest n.

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