

# Gov 50: 22. Uncertainty in Regressions

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Fall 2018

1. Today's agenda
2. Regression review
3. OLS as a estimator
4. Wrapping up

# 1/ Today's agenda

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  - ▶ We'll learn some key concepts for interpreting regression coefficients today.

## **2/** Regression review

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Name	Description
<code>shortnam</code>	three-letter country code
<code>africa</code>	indicator for if the country is in Africa
<code>avexpr</code>	strength of property rights (protection against expropriation)
<code>logpgp95</code>	log GDP per capita
<code>imr95</code>	infant mortality rate



# AJR scatterplot



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  - ▶ Population intercept:  $\beta_0$
  - ▶ Population slope:  $\beta_1$
- Error/disturbance:  $\varepsilon_i$ 
  - ▶ Represents all unobserved error factors influencing  $Y_i$  other than  $X_i$ .

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- Get these estimates by the **least squares method**.
- Minimize the **sum of the squared residuals** (SSR):

$$\text{SSR} = \sum_{i=1}^n \hat{\epsilon}_i^2 = \sum_{i=1}^n (Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i)^2$$

## 3/ OLS as a estimator

# Estimators

- Remember: least squares is an estimator—it's a machine that we plug data into and we get out estimates.

# Estimators

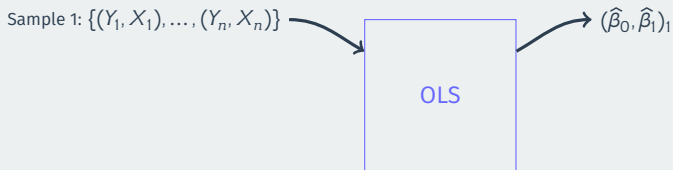
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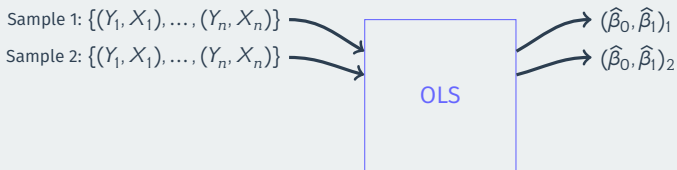
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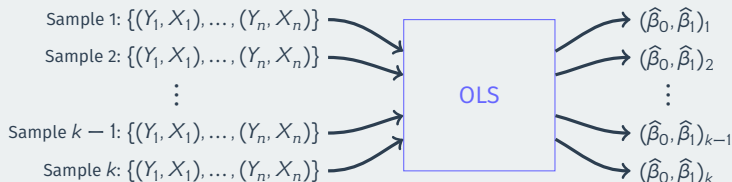
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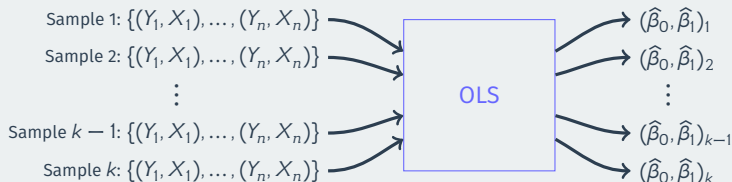
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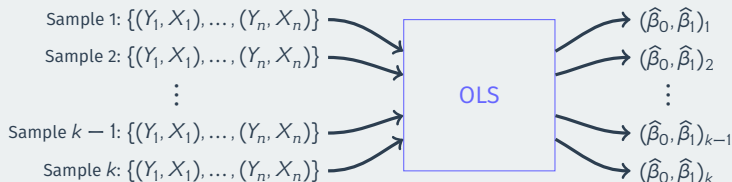


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- Just like the sample mean, sample difference in means, or the sample variance
- It has a sampling distribution, with a sampling variance/standard error, etc.

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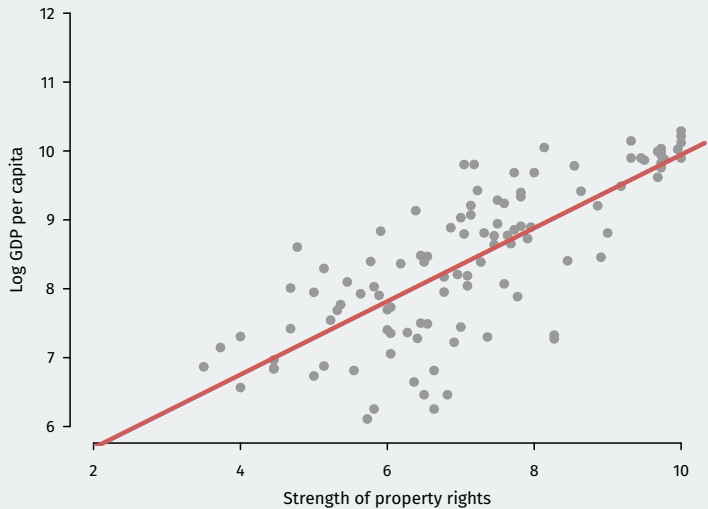
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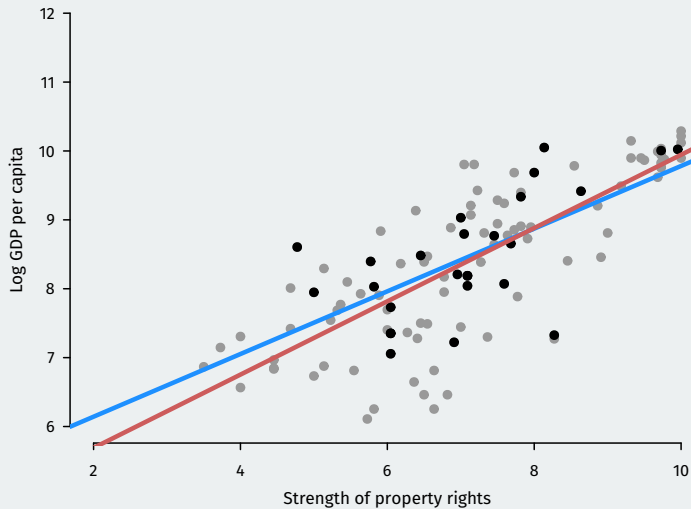
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- 3. Plot the estimated regression line

# Population regression

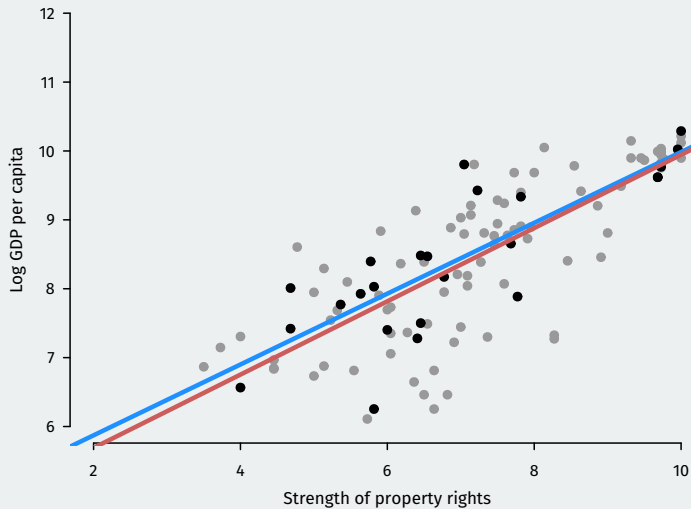




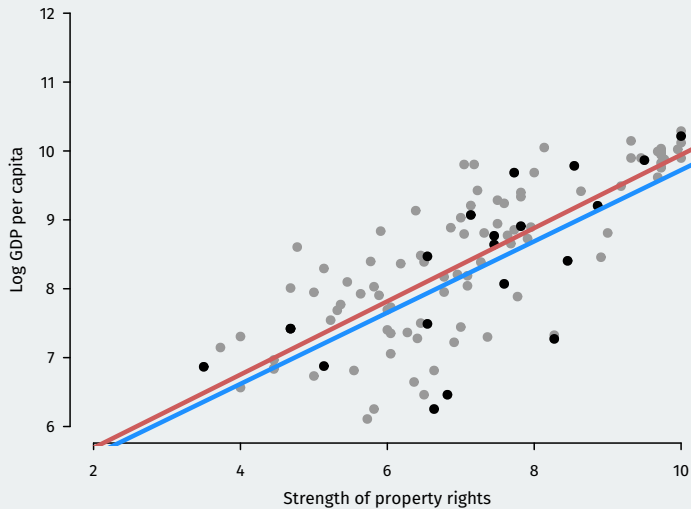
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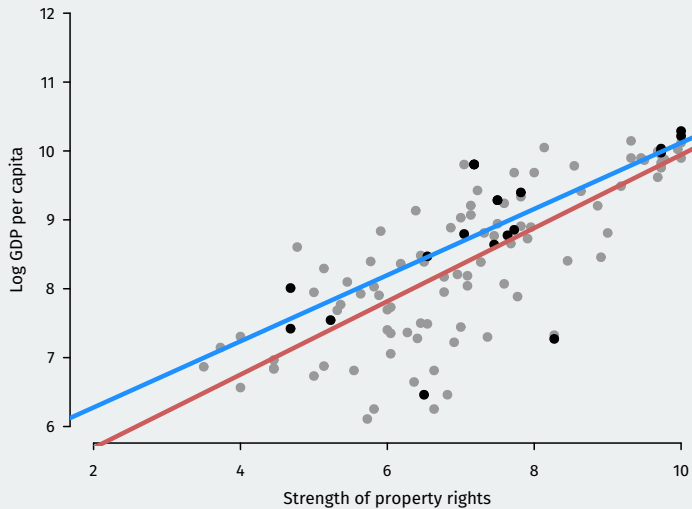
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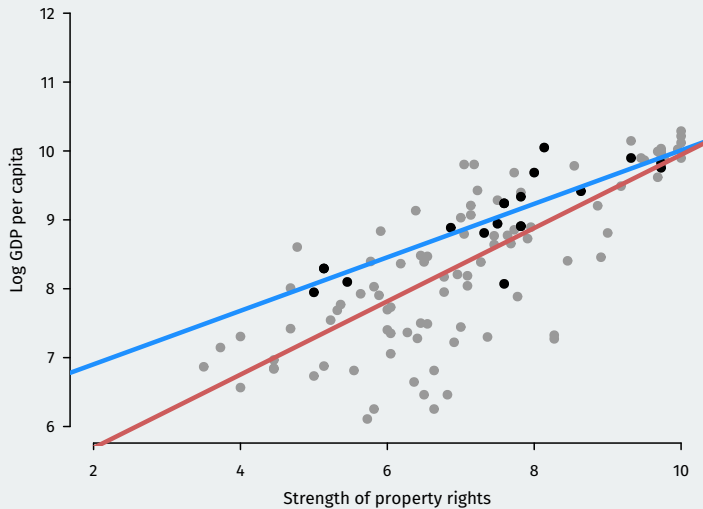
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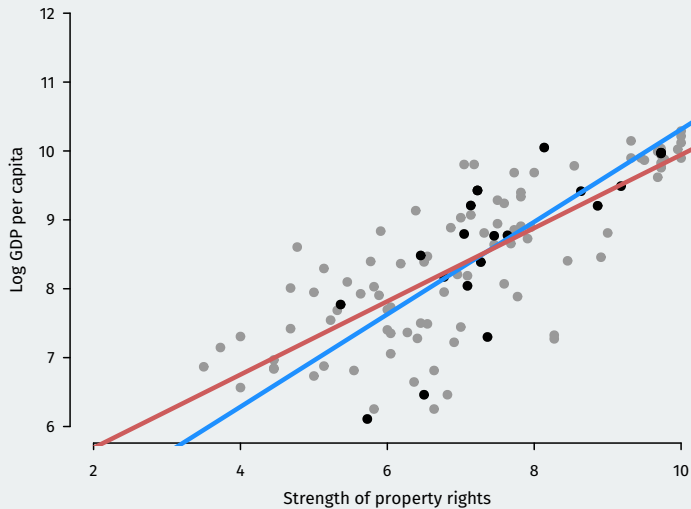
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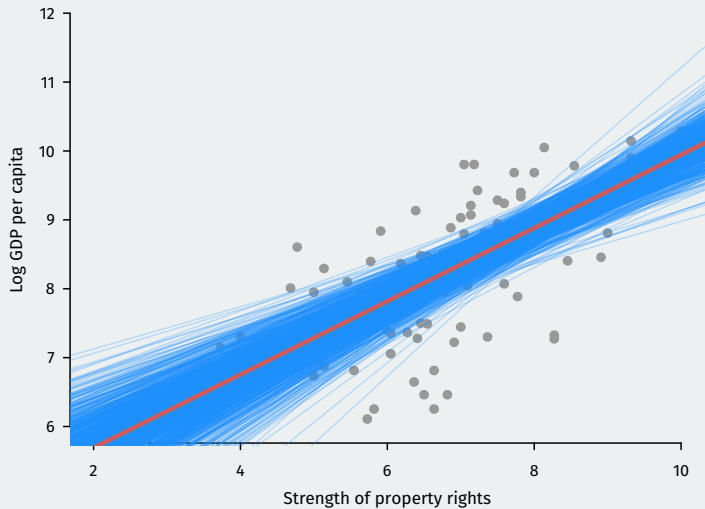
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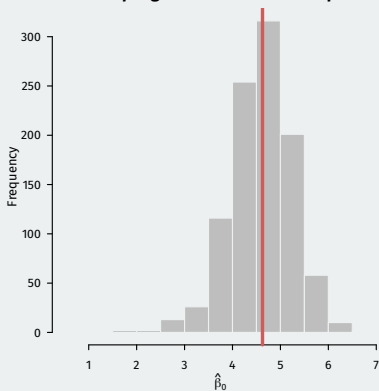
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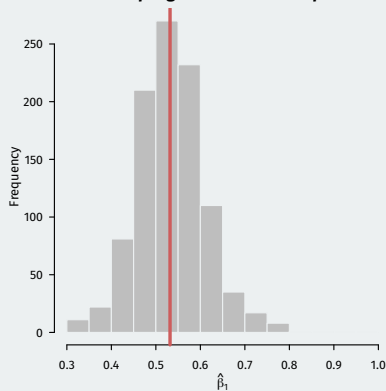
# Sampling distribution of OLS

- You can see that the estimated slopes and intercepts vary from sample to sample, but that the “average” of the lines looks about right.

**Sampling distribution of intercepts**



**Sampling distribution of slopes**





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  - ▶  $\hat{\beta}_1$  is **statistically significant** if its p-value from this test is below some threshold (usually 0.05)

```
ajr.reg <- lm(logpgp95 ~ avexpr, data = ajr)
summary(ajr.reg)
```

```
##
## Call:
## lm(formula = logpgp95 ~ avexpr, data = ajr)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.902 -0.316  0.138  0.422  1.441
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)   4.6261     0.3006   15.4   <2e-16 ***
## avexpr        0.5319     0.0406   13.1   <2e-16 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.718 on 109 degrees of freedom
## (52 observations deleted due to missingness)
## Multiple R-squared:  0.611, Adjusted R-squared:  0.608
## F-statistic: 171 on 1 and 109 DF, p-value: <2e-16
```



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- Inference:
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$$Y_i = \beta_0 + \beta_1 X_{i1} + \beta_2 X_{i2} + \dots + \beta_p X_{ip} + \epsilon_i$$

- Interpretation of  $\beta_j$ : an increase in the outcome associated with a one-unit increase in  $X_{ij}$  when other variables don't change their values
- Inference:
  - ▶ Confidence intervals constructed exactly the same for  $\hat{\beta}_j$
  - ▶ Hypothesis tests done exactly the same for  $\hat{\beta}_j$

# Multiple regression

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  - ▶  $\rightsquigarrow$  interpret p-values the same as before.



```
ajr.reg <- lm(logpgp95 ~ avexpr + africa + imr95, data = ajr)
summary(ajr.reg)
```

```
##
## Call:
## lm(formula = logpgp95 ~ avexpr + africa + imr95, data = ajr)
##
## Residuals:
##      Min       1Q   Median       3Q      Max
## -1.3928 -0.2708  0.0865  0.2749  1.1652
##
## Coefficients:
##              Estimate Std. Error t value Pr(>|t|)
## (Intercept)  7.01362    0.40445   17.34 < 2e-16 ***
## avexpr       0.28872    0.05046    5.72 0.00000043 ***
## africa      -0.02069    0.18622   -0.11    0.91
## imr95       -0.01549    0.00271   -5.71 0.00000045 ***
## ---
## Signif. codes:
## 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 0.492 on 56 degrees of freedom
## (103 observations deleted due to missingness)
## Multiple R-squared:  0.778, Adjusted R-squared:  0.766
## F-statistic: 65.4 on 3 and 56 DF, p-value: <2e-16
```

# Regression tables

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- Standard errors, p-values, sample size, and  $R^2$  may be reported as well.

TABLE 2—OLS REGRESSIONS

	Whole world (1)	Base sample (2)	Whole world (3)	Whole world (4)	Base sample (5)	Base sample (6)	Whole world (7)	Base sample (8)
	Dependent variable is log GDP per capita in 1995						Dependent variable is log output per worker in 1988	
Average protection against expropriation risk, 1985–1995	0.54 (0.04)	0.52 (0.06)	0.47 (0.06)	0.43 (0.05)	0.47 (0.06)	0.41 (0.06)	0.45 (0.04)	0.46 (0.06)
Latitude			0.89 (0.49)	0.37 (0.51)	1.60 (0.70)	0.92 (0.63)		
Asia dummy				-0.62 (0.19)		-0.60 (0.23)		
Africa dummy				-1.00 (0.15)		-0.90 (0.17)		
“Other” continent dummy				-0.25 (0.20)		-0.04 (0.32)		
$R^2$	0.62	0.54	0.63	0.73	0.56	0.69	0.55	0.49
Number of observations	110	64	110	110	64	64	108	61

## 4/ Wrapping up

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  - ▶ Be skeptical of causal claims unless groups are really comparable.
  - ▶ Think carefully about sampling biases when people make claims.

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- Outside classes:
  - ▶ Work with faculty on research projects!

# Thanks!

Thanks for a really fun and engaging semester! Good luck with your final projects!