Gov 50: 17. Sums and Means in Large Samples

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1/ Today's agenda

Logistics

- HW 4 due Thursday.

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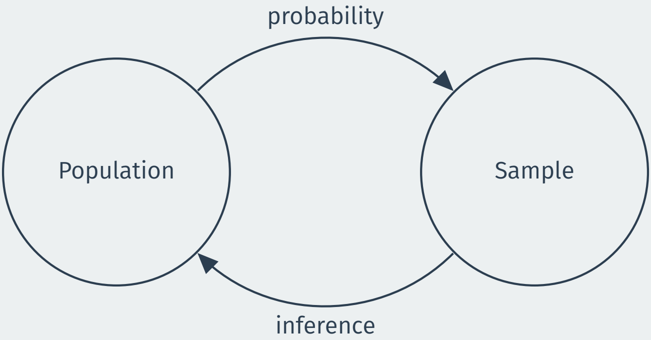
  - Law of large numbers
  - Central limit theorem

Learning about populations



- **Probability**: formalize the uncertainty about how our data came to be.
- **Inference**: learning about the population from a set of data.

2/ Sample means

Fulton county data

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Name	Description
`turnout`	did person vote (1) or not (0) in 1994?
`black`	is this person black (1) or not (0)?
`sex`	is this person a woman (1) or not (0)?
`age`	age
`dem`	is this person registered as a Democrat (1) or not (0)?
`rep`	is this person registered as a Republican (1) or not (0)?
`urban`	registered in a city (1) or not (0)?

Load Fulton county data

```
fulton <- read.csv("data/fulton.csv")
head(fulton)
```

```
##   turnout black sex age dem rep urban
## 1       0     0   1  19   0   0     0
## 2       0     0   0  35   0   0     0
## 3       0     1   0  36   0   0     1
## 4       1     0   0  27   0   0     1
## 5       1     1   1  79   1   0     1
## 6       1     0   1  42   1   0     0
```

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- **Asymptotics**: what can we learn as n gets big?

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- This is the average age of two randomly selected respondents.

Distribution of sums/means

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draw 1	49	29		

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- Fulton County data:
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  - Will the average age in the sample be close to the population age?

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Mean and variance of the sample mean

Suppose that $X_1, \ldots, X_n$ are i.i.d. r.v.s with $\mathbb{E}[X_i] = \mu$ and $\mathbb{V}[X_i] = \sigma^2$. Then:

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  - Variance of $\overline{X}_n$ depends on the population variance of X_i and the sample size
- Standard deviation of the sample mean is called its **standard error**:

$$SE = \sqrt{\mathbb{V}[\overline{X}_n]} = \frac{\sigma}{\sqrt{n}}$$

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- The distribution of sample mean "collapses" to population mean.

LLN by simulation in R

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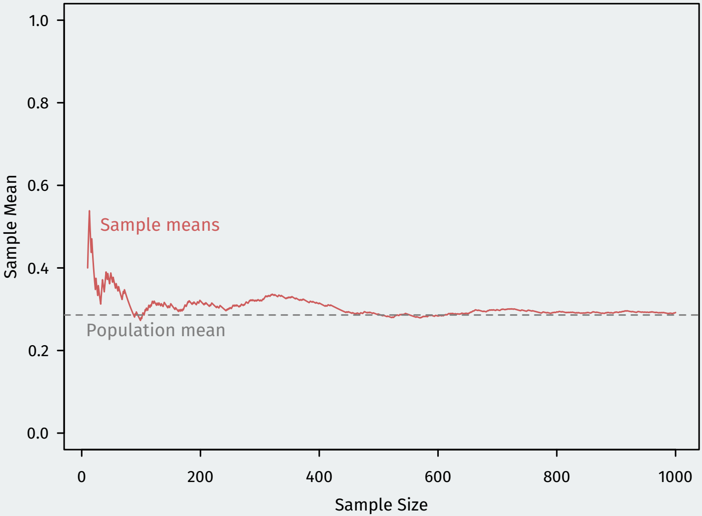
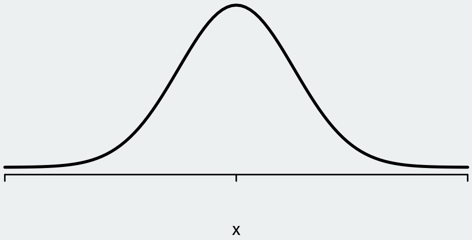
```
dem.mean <- mean(fulton$dem)
sims <- 1000

# draw a random sample of row numbers (with replacement)
samp <- sample(1:nrow(fulton), size = sims, replace = TRUE)
dem.samp <- fulton$dem[samp]

# calculate the mean of the first i values
samp.means <- rep(NA, times = sims)
for (i in 1:sims) {
  samp.means[i] <- sum(dem.samp[1:i]) / i
}
```

LLN in action



3/ Normal distribution

Normal r.v.



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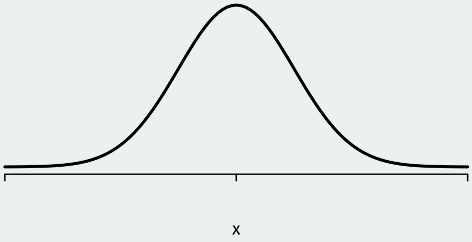

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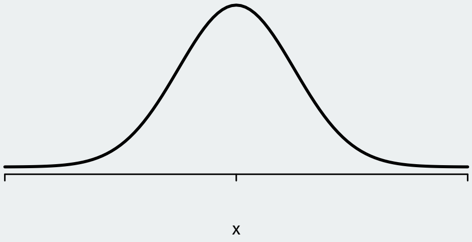

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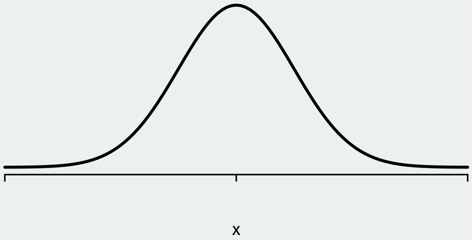

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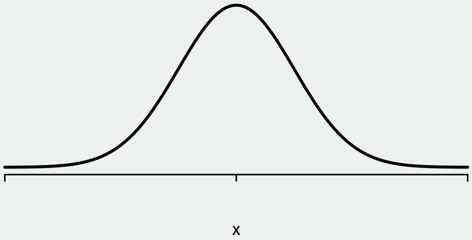

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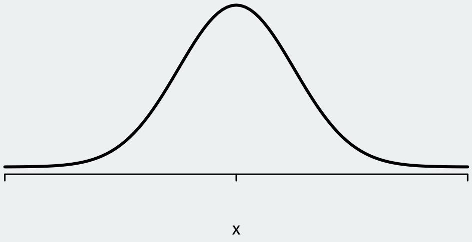

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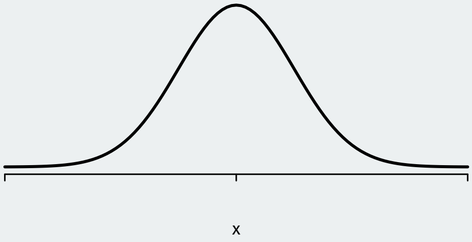

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  - **Everywhere positive**: any real value can possibly occur.

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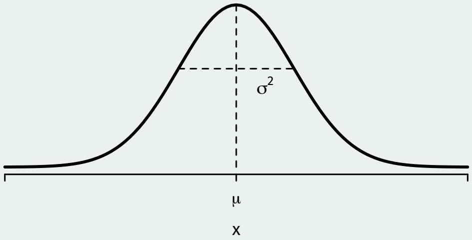

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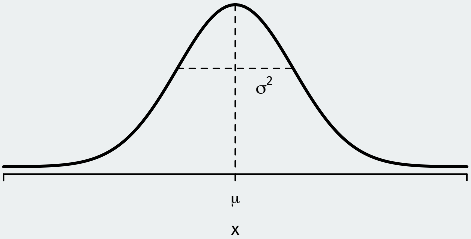

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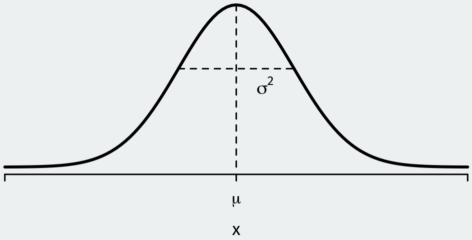

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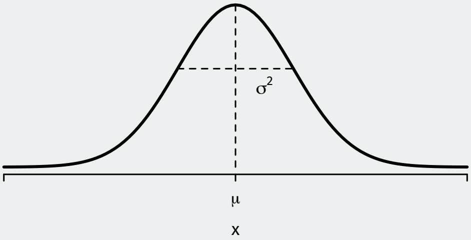

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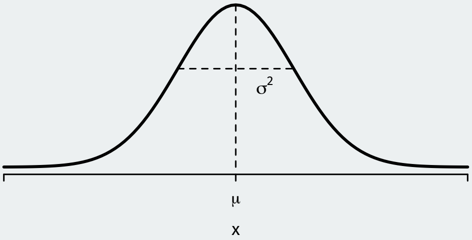

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- **Standard normal distribution**: mean 0 and standard deviation 1.

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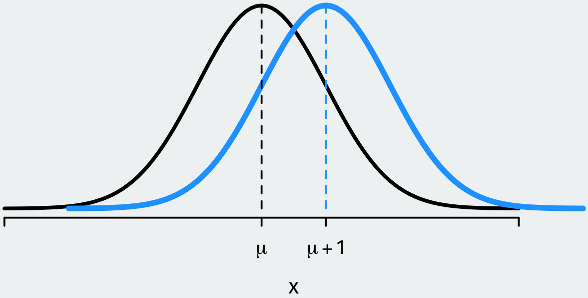
- Intuition: adding a constant to a normal shifts the distribution by that constant.



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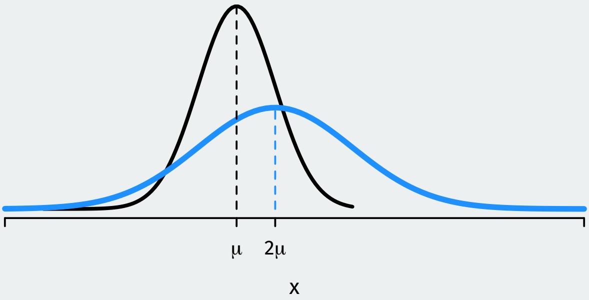

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- Subtract the mean and divide by the SD $\rightsquigarrow$ standard normal.
- z-score measures how many SDs away from the mean a value of X is.

4/ Central limit theorem

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Let $X_1, \ldots, X_n$ be i.i.d. r.v.s from a distribution with mean μ and variance σ^2. Then, $\overline{X}_n$ will be approximately distributed $N(\mu, \sigma^2/n)$ in large samples.

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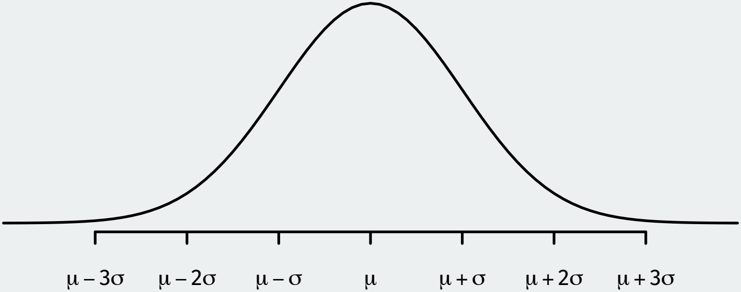
- "Sample means tend to be normally distributed as samples get large."
- $\rightsquigarrow$ we know how far away $\overline{X}_n$ will be from its mean.

Empirical Rule for the Normal Distribution



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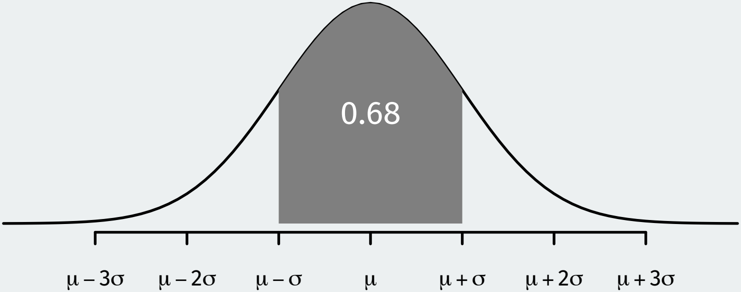

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  - $\approx$ 68% of the distribution of X is within 1 SD of the mean.

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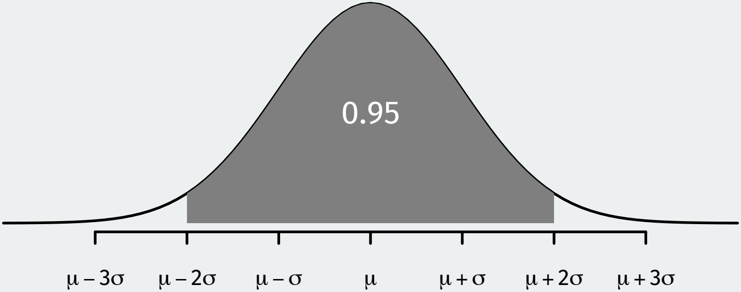

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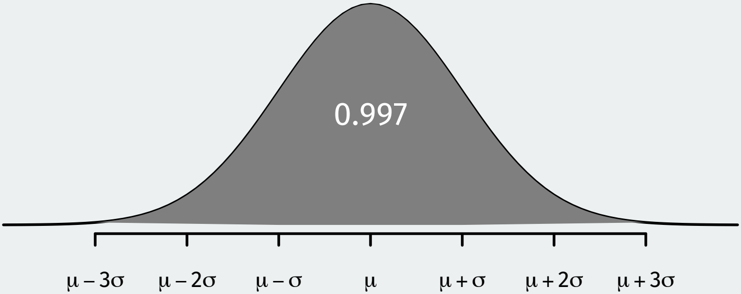

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  - $\approx$ 68% of the distribution of X is within 1 SD of the mean.
  - $\approx$ 95% of the distribution of X is within 2 SDs of the mean.
  - $\approx$ 99.7% of the distribution of X is within 3 SDs of the mean.

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- By CLT, sample mean $\approx$ normal with mean μ and SD $\frac{\sigma}{\sqrt{n}}$.

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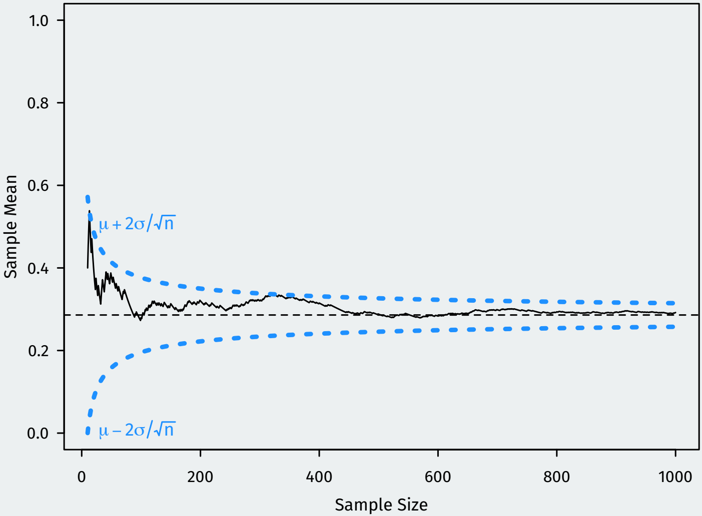
- By CLT, sample mean $\approx$ normal with mean μ and SD $\frac{\sigma}{\sqrt{n}}$.
- By empirical rule, sample mean will be within $2 \times \frac{\sigma}{\sqrt{n}}$ of the population mean 95% of the time.

CLT in action



CLT simulation

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3. Save the sample mean.
4. Repeat steps 1-3 a large number of times.

CLT in action

```r
dem.sigma <- sd(fulton$dem)
n <- 1000
sims <- 5000
dem.means <- rep(NA, times = sims)
for (i in 1:sims) {
  ## take i.i.d. sample of row numbers
  samp.ind <- sample(1:nrow(fulton), size = n,
                     replace = TRUE)

  ## get the values of "dem" for sample
  dem.sample <- fulton$dem[samp.ind]

  ## record mean of this sample
  dem.means[i] <- mean(dem.sample)
}
```

```r
## mean and sd of the sample means from each
## repeated sample
mean(dem.means)
```

```
## [1] 0.286
```

```r
sd(dem.means)
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## [1] 0.0142
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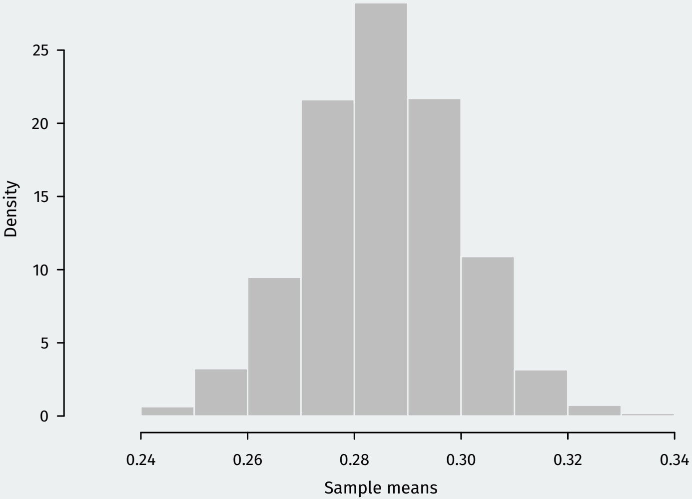
```r
## compare to what the CLT predicts from population
mean(fulton$dem)
```

```
## [1] 0.286
```

```r
sd(fulton$dem)/sqrt(n)
```

```
## [1] 0.0143
```

Histogram of sample means



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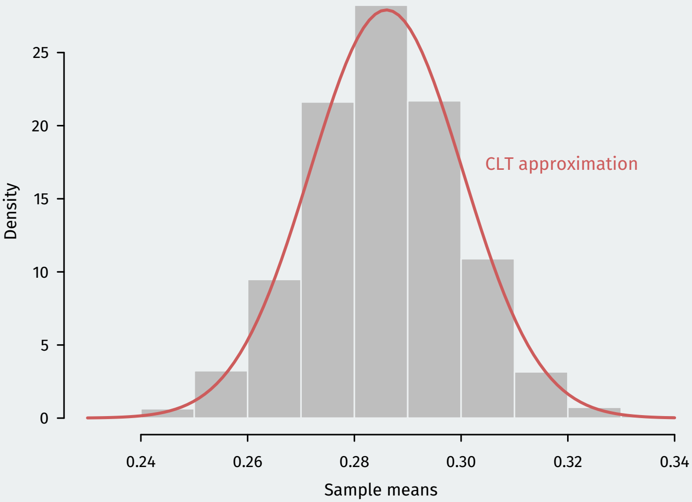

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- Why do we care about LLN/CLT?
  - CLT gives us assurances our 1 sample mean will won't be too far from population mean.
  - CLT will also help us create measure of uncertainty for our estimates.

Next time

- Today: learning about samples given population information

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- Next: Learning about population values from the sample.