Gov 50: 20. Hypothesis testing: One-sample Tests

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1. Today's agenda

2. Statistical thought experiments

3. Hypothesis tests

4. p-values

5. Small samples

1/ Today's agenda

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  - Midterm 2: definitely harder, but remember the curve!
  - You all are learning a ton in this class and it shows.

2/ Statistical thought experiments

The lady tasting tea

Your friend asks you to grab a tea with milk for her before meeting up and she says that she prefers tea poured before the milk. You stop by Tealuxe and ask for a tea with milk. When you bring it to her, she complains that it was prepared milk-first.

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  - Present cups to friend in a **random** order
  - Ask friend to pick which 4 of the 8 were milk-first.

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- $\rightsquigarrow$ the guessing hypothesis might be implausible.

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  - A recent poll of 1600 registered voters found that 43% of the public approves of Donald Trump.
  - Has support for Trump changed or could this poll be just due to random chance?

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- This procedure is general, but we'll focus on tests of a single population mean today.

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- **Probabilistic** proof by contradiction: try to disprove the null.

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  - Poll has $\overline{X} = 0.43$ with $n = 1600$.

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  - How many SEs away from the null guess is the sample mean?

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- Estimate the test statistic:

$$Z_{\text{obs}} = \frac{\overline{X} - p}{\sqrt{p(1-p)/n}} = \frac{0.43 - 0.46}{\sqrt{0.46(1-0.46)/1600}} \approx -2.4$$

4/ p-values

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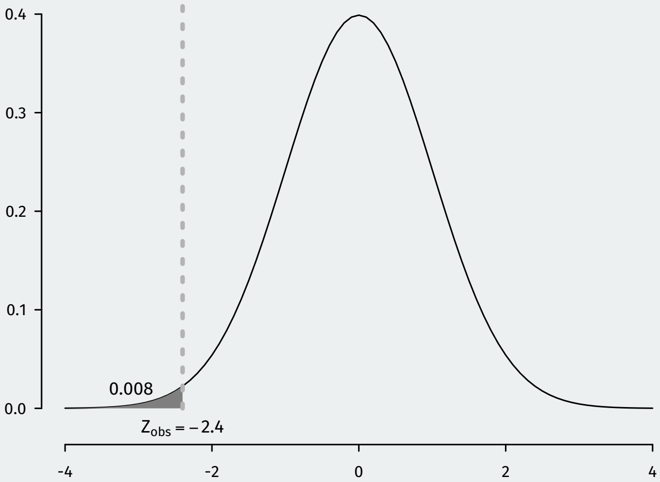
- How to calculate?
  - We know Z is distributed standard normal $\rightsquigarrow$ use R!

Standard normal probabilities in R

- The `pnorm(x)` function will give the probability of being less x in a standard normal:

```
pnorm(-2.4)
```

```
## [1] 0.0082
```


0.4
0.3
0.2
0.1
0.0
0.008
$Z_{obs} = -2.4$
-4
-2
0
2
4

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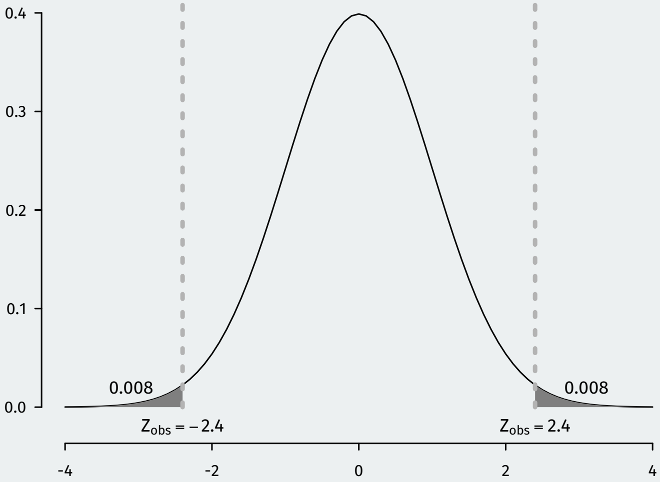
- Two sided p-value: $\mathbb{P}(Z > |Z_{\text{obs}}|) + \mathbb{P}(Z < -|Z_{\text{obs}}|)$
  - Because of the symmetry of the normal: $2 \times \mathbb{P}(Z > |Z_{\text{obs}}|)$



- two-sided p-value: 0.016

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  - Smaller p-values $\rightsquigarrow$ more evidence against the null.

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  - $p \geq 0.1$ "not statistically significant"
  - $p < 0.05$ "statistically significant"

Rejecting the null

- **Step 5**: use p-value to decide whether to reject null or not.
- Choose a threshold below which you'll reject the null.
  - **Test level** α: the threshold for a test.
  - Decision rule: "reject the null if the p-value is below α"
  - Otherwise "fail to reject" or "retain", not "accept the null"
- Common (arbitrary) thresholds:
  - $p \geq 0.1$ "not statistically significant"
  - $p < 0.05$ "statistically significant"
  - $p < 0.01$ "highly significant"

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- Type II error less serious (missed out on an awesome finding)

5/ Small samples

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- Very difficult to get around this problem without more information.

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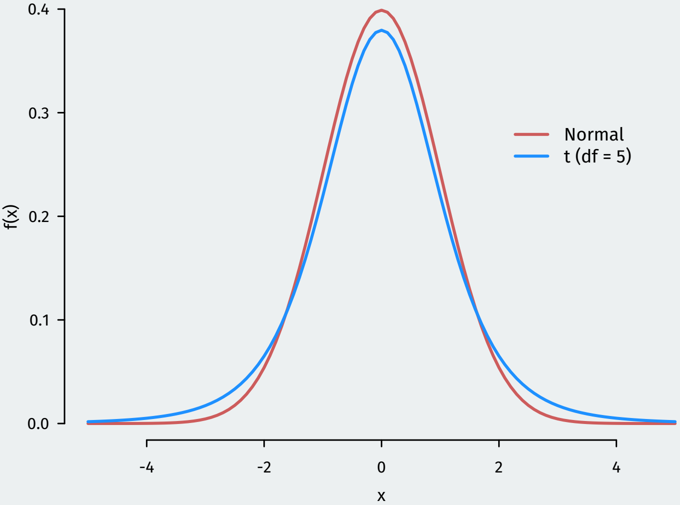
  - Similar to normal with fatter tails $\rightsquigarrow$ higher likelihood of extreme events.

Who was Student?

Student's t distribution



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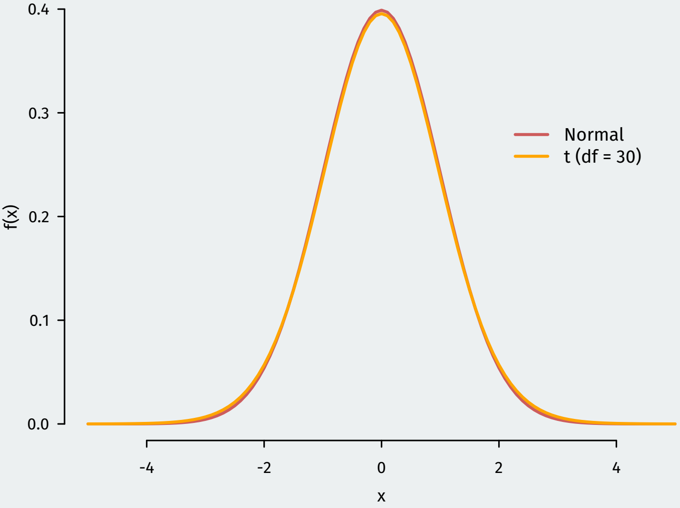

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- R will almost always calculate p-values for you, so details of t-distribution aren't massively important.

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- Next week: inference for regression.