How to Make Causal Inferences with Time-Series Cross-Sectional Data

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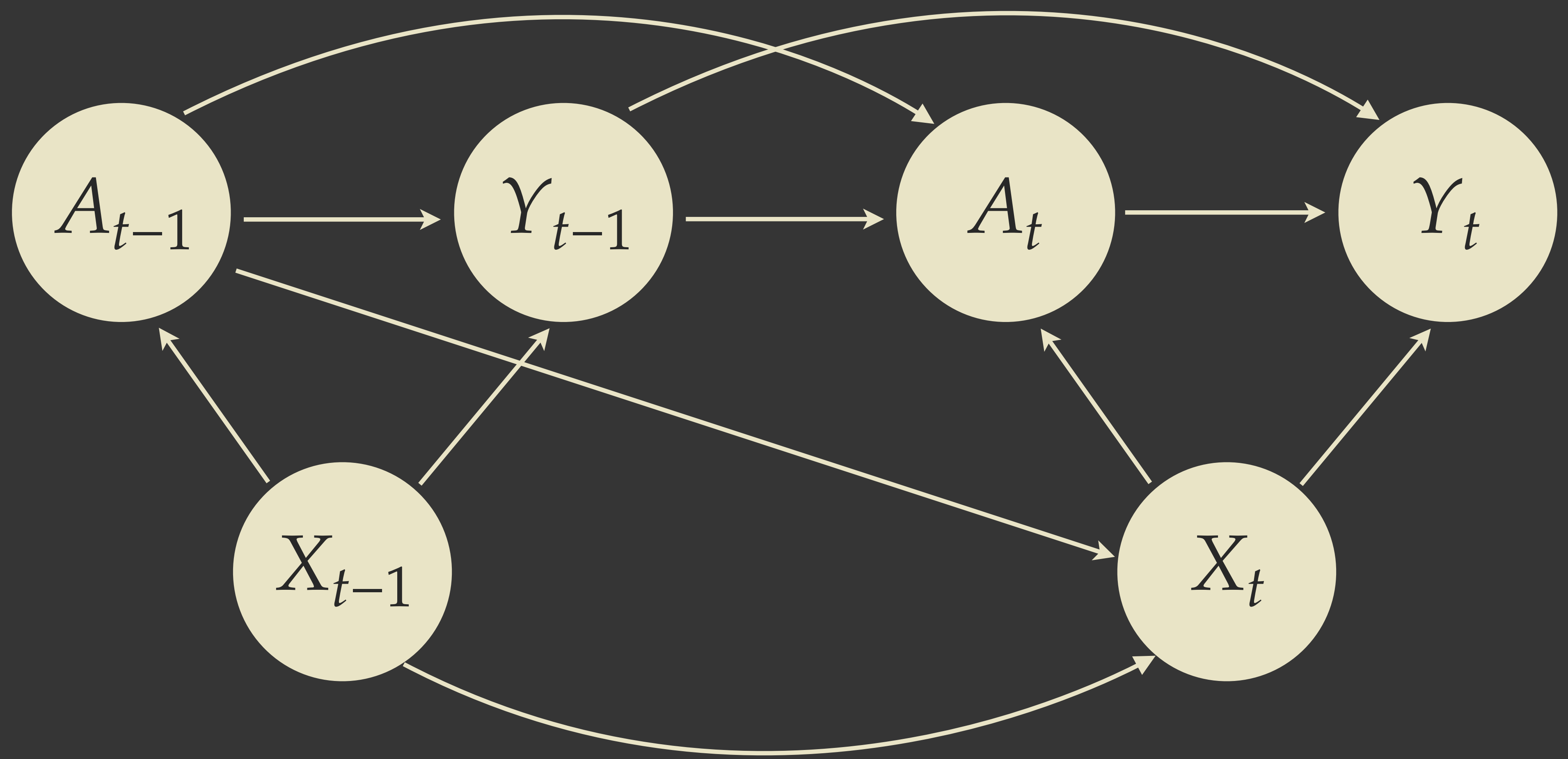
How to Make Causal Inferences with Time-Series Cross-Sectional Data

How to Make Causal Inferences with Time-Series Cross-Sectional Data

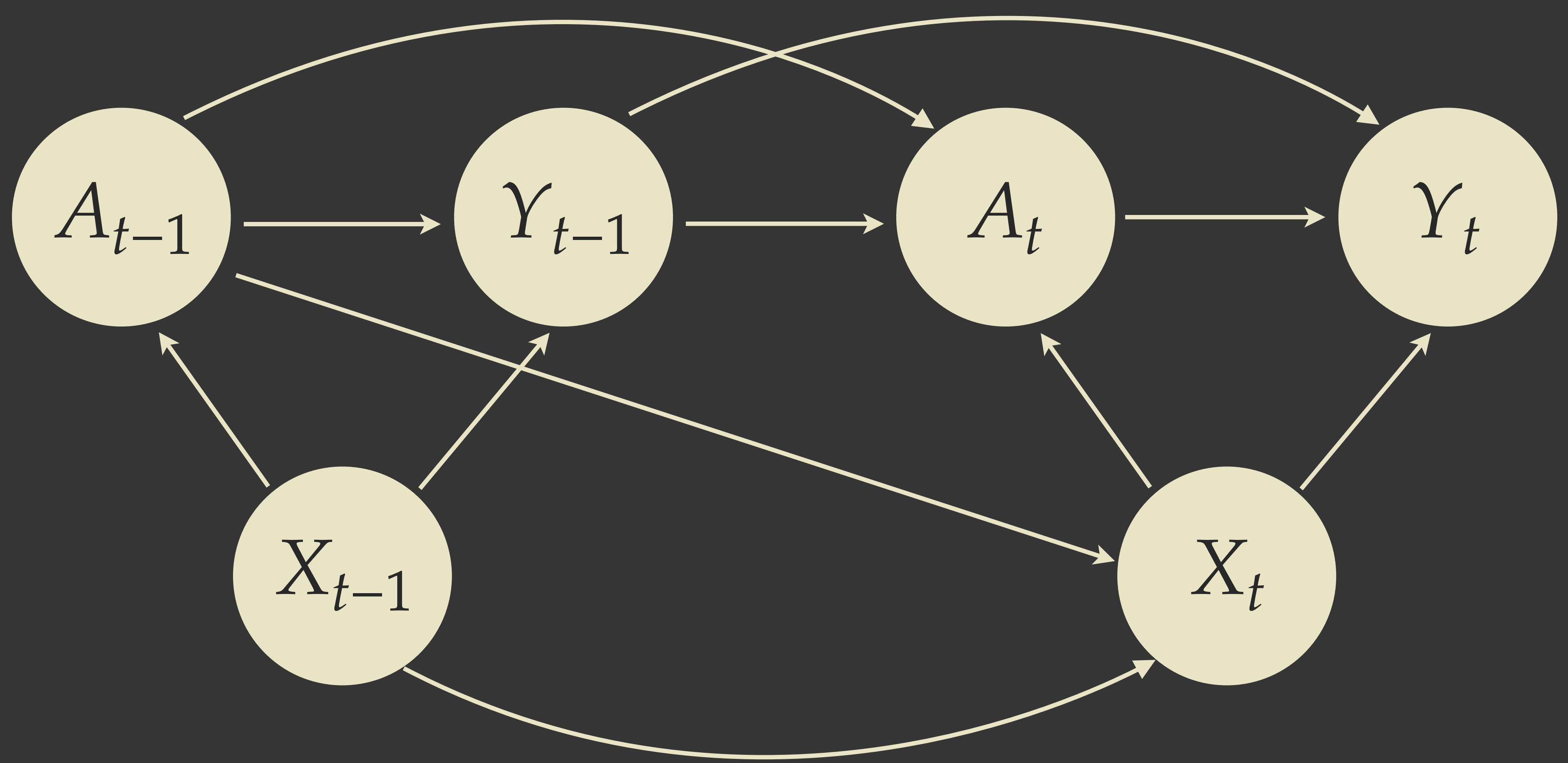
Very Carefully.

How to Make Causal Inferences with Time-Series Cross-Sectional Data

Using weights.

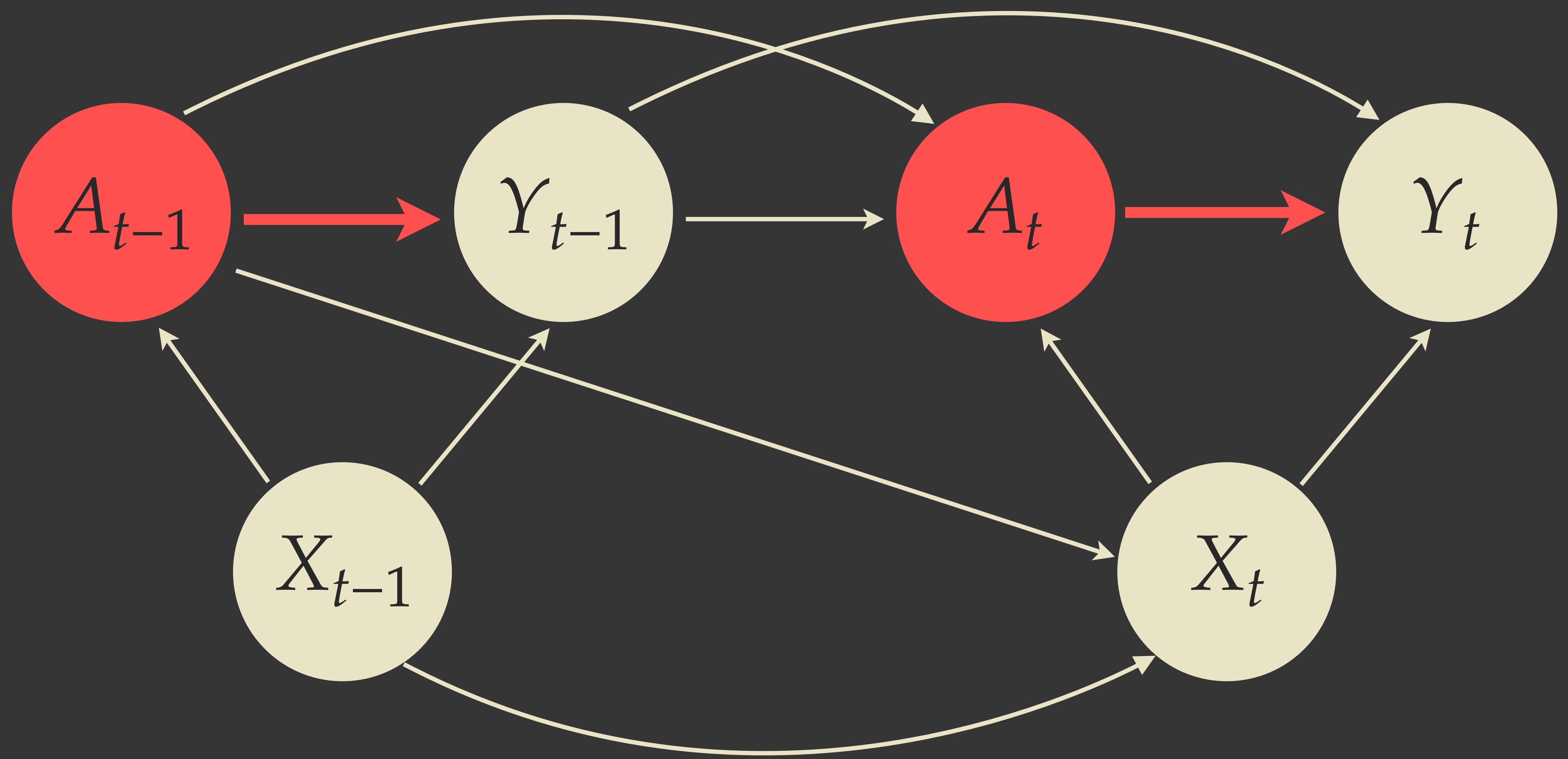

A_{t-1}
Y_{t-1}
A_t
Y_t
X_{t-1}
X_t

What is the effect of A on Y?



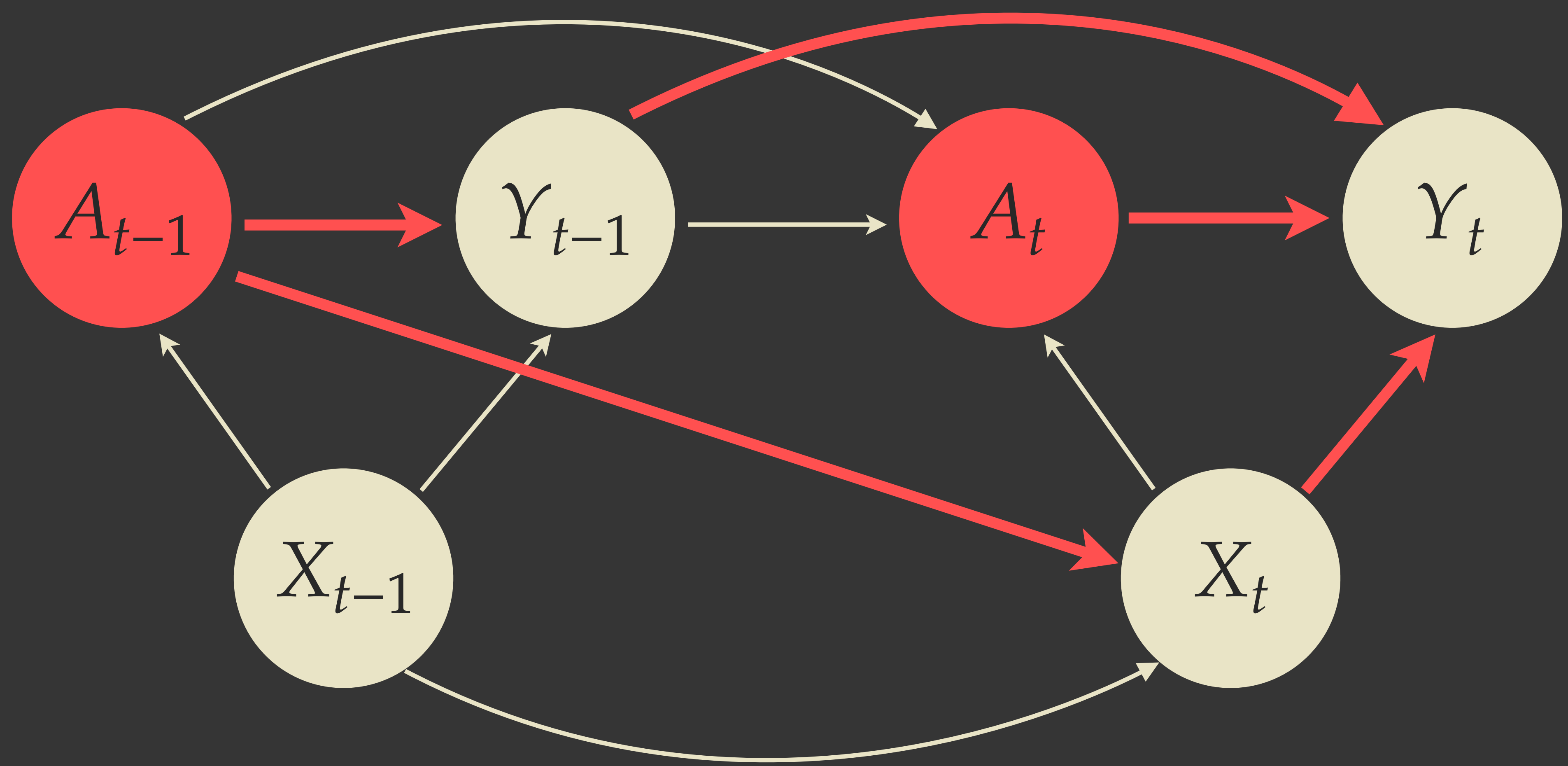
What is the effect of A on Y?

contemporaneous



What is the effect of A on Y?

treatment history



Shouldn't we have more notation?

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$$\underline{A}_t = (A_1, \dots, A_t)$$

Treatment history

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Treatment history

$$\underline{a}_t = (a_1, \dots, a_t)$$

Specific instance of a treatment history

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$$\underline{A}_t = (A_1, \dots, A_t)$$

Treatment history

$$\underline{a}_t = (a_1, \dots, a_t)$$

Specific instance of a treatment history

$$Y_t(\underline{a}_t)$$

Potential outcomes

The effect of history

The effect of history

Average Treatment History Effect

$$\tau(\underline{a}_t, \underline{a}'_t) = E[Y_t(\underline{a}_t) - Y_t(\underline{a}'_t)]$$

The effect of history

Average Treatment History Effect

ATHE

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$$\tau(\underline{a}_t, \underline{a}'_t) = E[Y_t(\underline{a}_t) - Y_t(\underline{a}'_t)]$$

1 1 1 1 1 1 1

VS

0 0 0 0 0 0 0

The effect of history

The effect of history

Blip Effect $\tau_b(\underline{a}_{t-1}) = E[Y_t(\underline{a}_{t-1}, 1) - Y_t(\underline{a}_{t-1}, 0)]$

The effect of history

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0 0 0 0 0 0 1

vs

0 0 0 0 0 0 0

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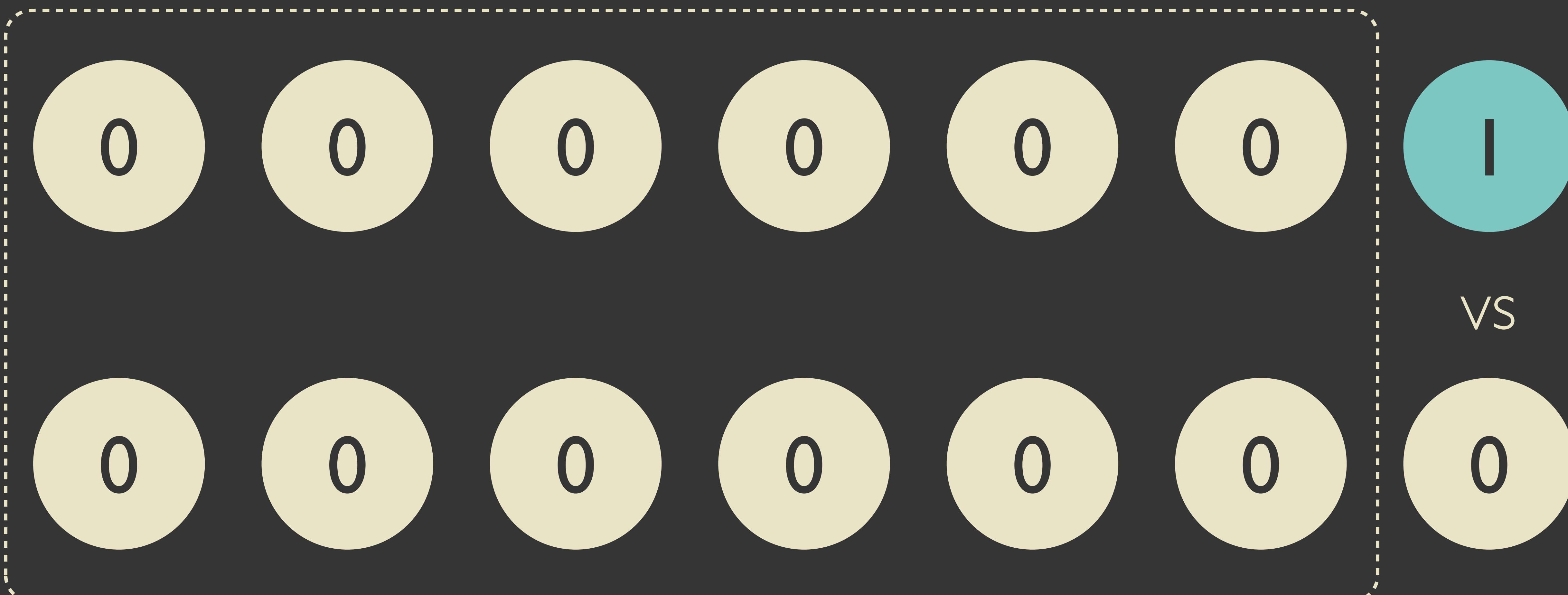

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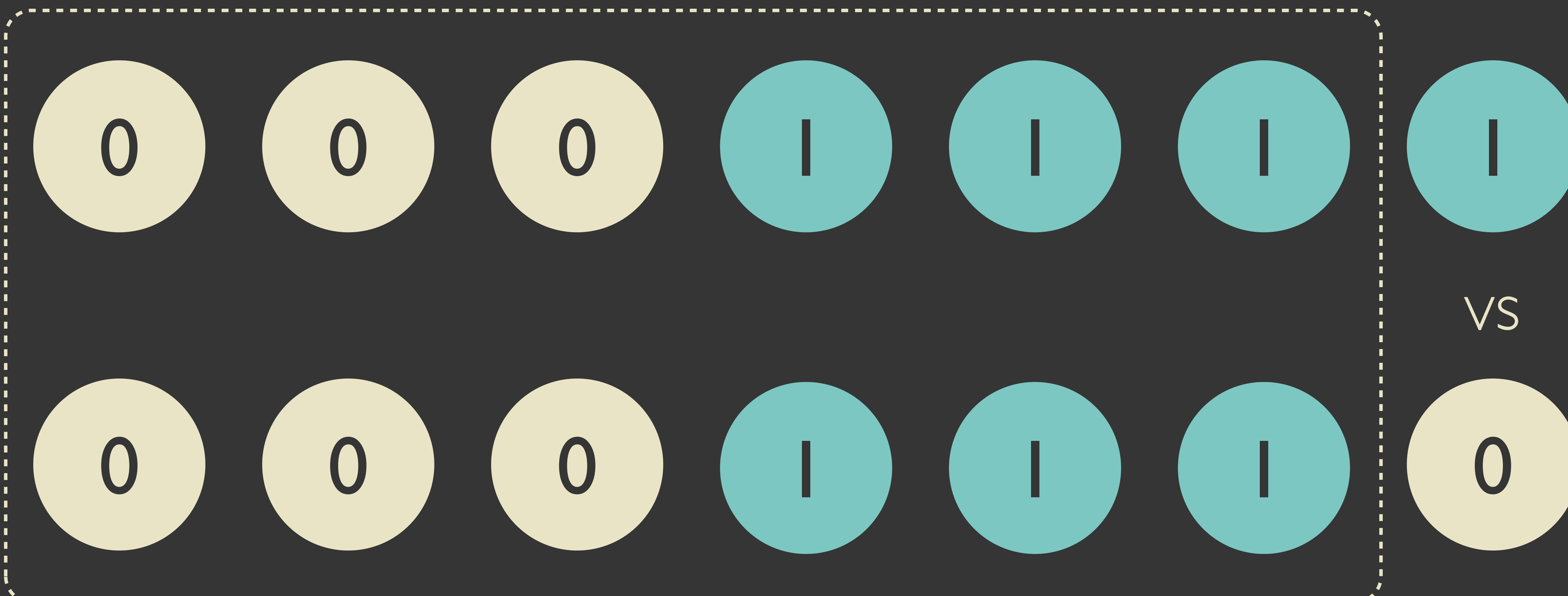

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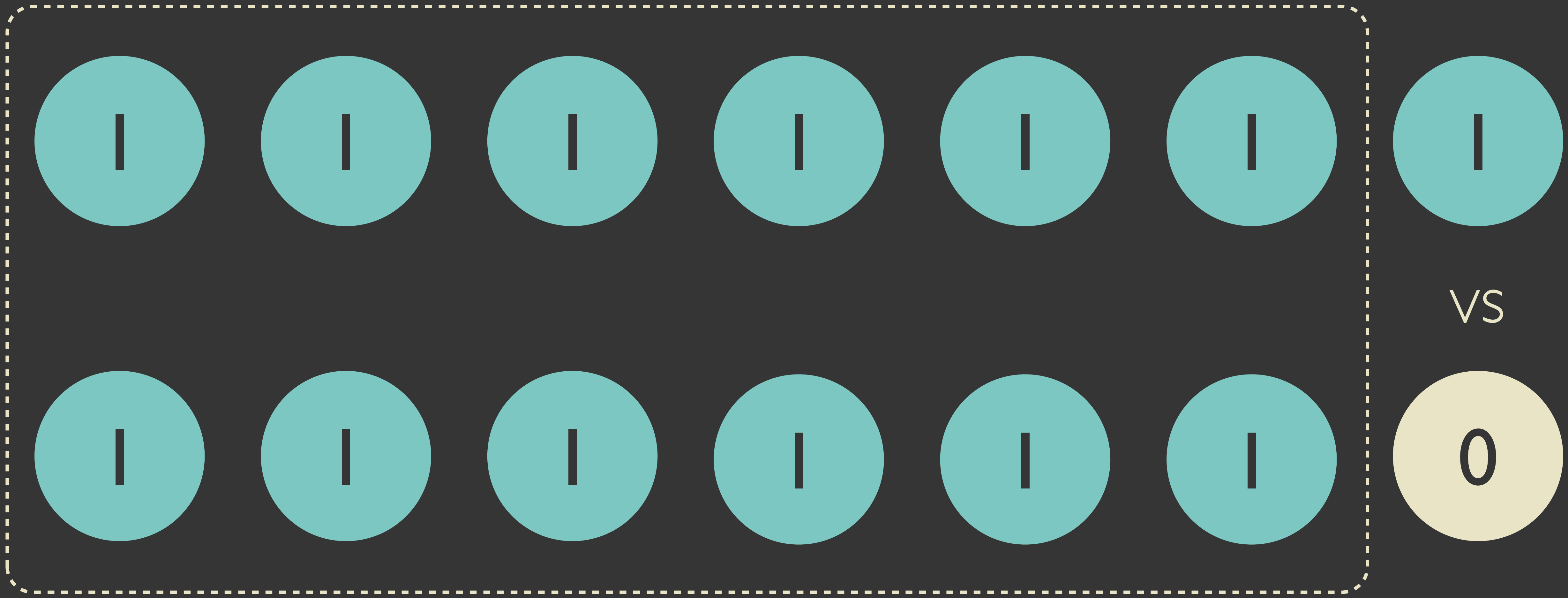

The effect of history

The effect of history

Contemporaneous
Effect of Treatment

$$\tau_t = E[\tau_b(\underline{a}_{t-1})]$$

The effect of history

Contemporaneous
Effect of Treatment

CET

$$\tau_t = E[\tau_b(\underline{a}_{t-1})]$$

The effect of history

Contemporaneous
Effect of Treatment

CET

$$\tau_t = E[\tau_b(\underline{a}_{t-1})]$$

1

vs

0

The effect of history

Contemporaneous
Effect of Treatment

CET

$$\tau_t = E[\tau_b(\underline{a}_{t-1})]$$

Marginalize over the past

1

vs

0

TSCS data under sequential ignorability

$$Y_t(\underline{a}_t) \perp\!\!\!\perp A_t | \underline{X}_t, \underline{Y}_{t-1}, \underline{A}_{i,t-1} = \underline{a}_{t-1}$$

Treatment is unrelated
to the potential outcomes

...conditional on the
covariate history.

How conditioning leads you astray

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...for some questions.

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$$Y_t = \beta_0 + \beta_1 A_t + \beta_2 X_t + \beta_3 Y_{t-1} + \beta_4 A_{t-1}$$

How conditioning leads you astray

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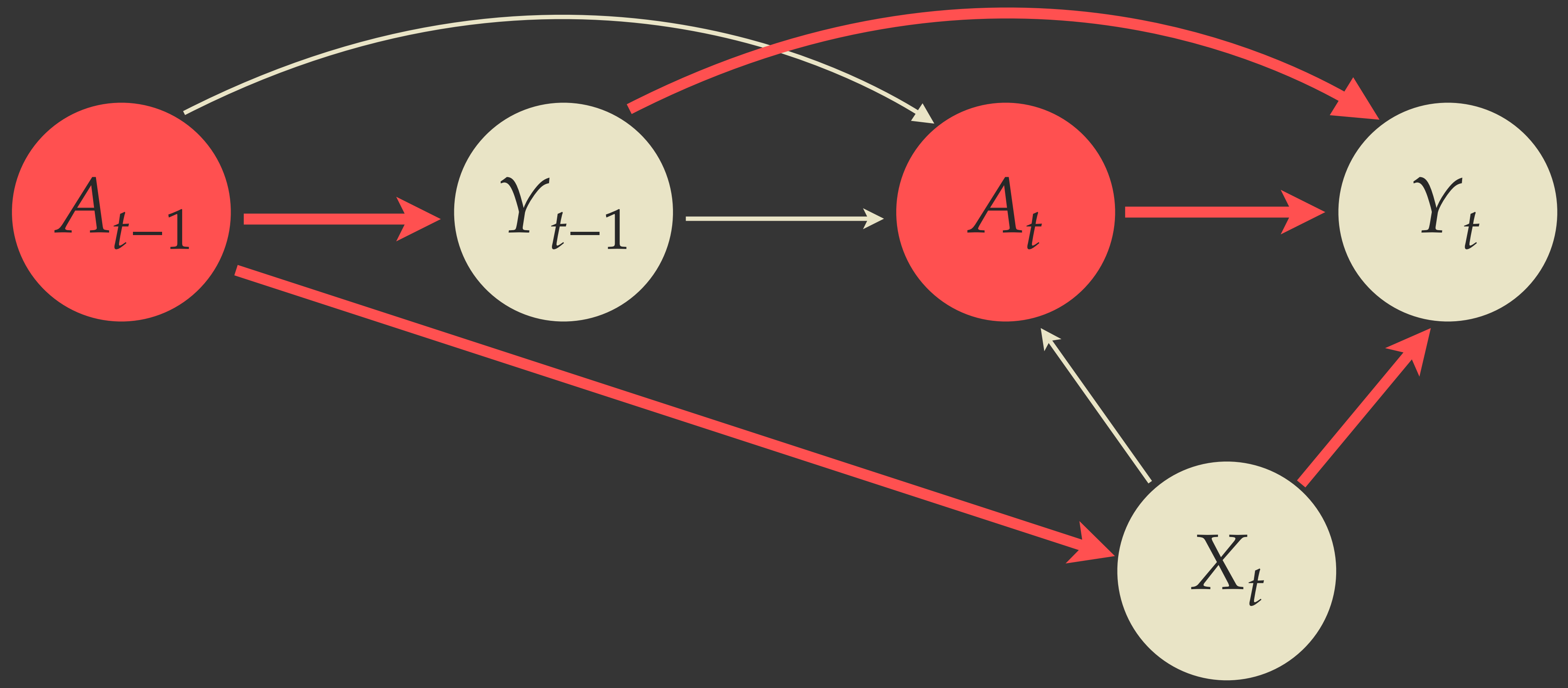

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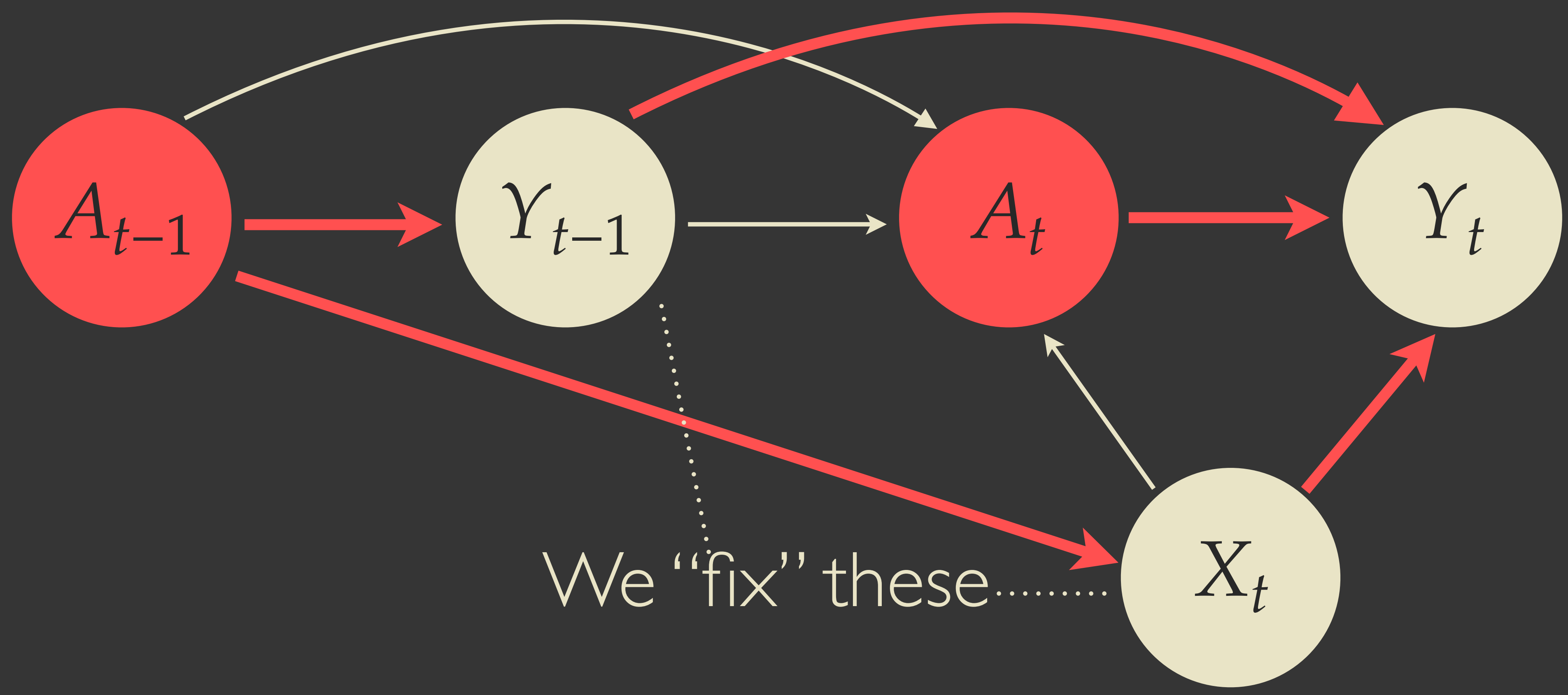

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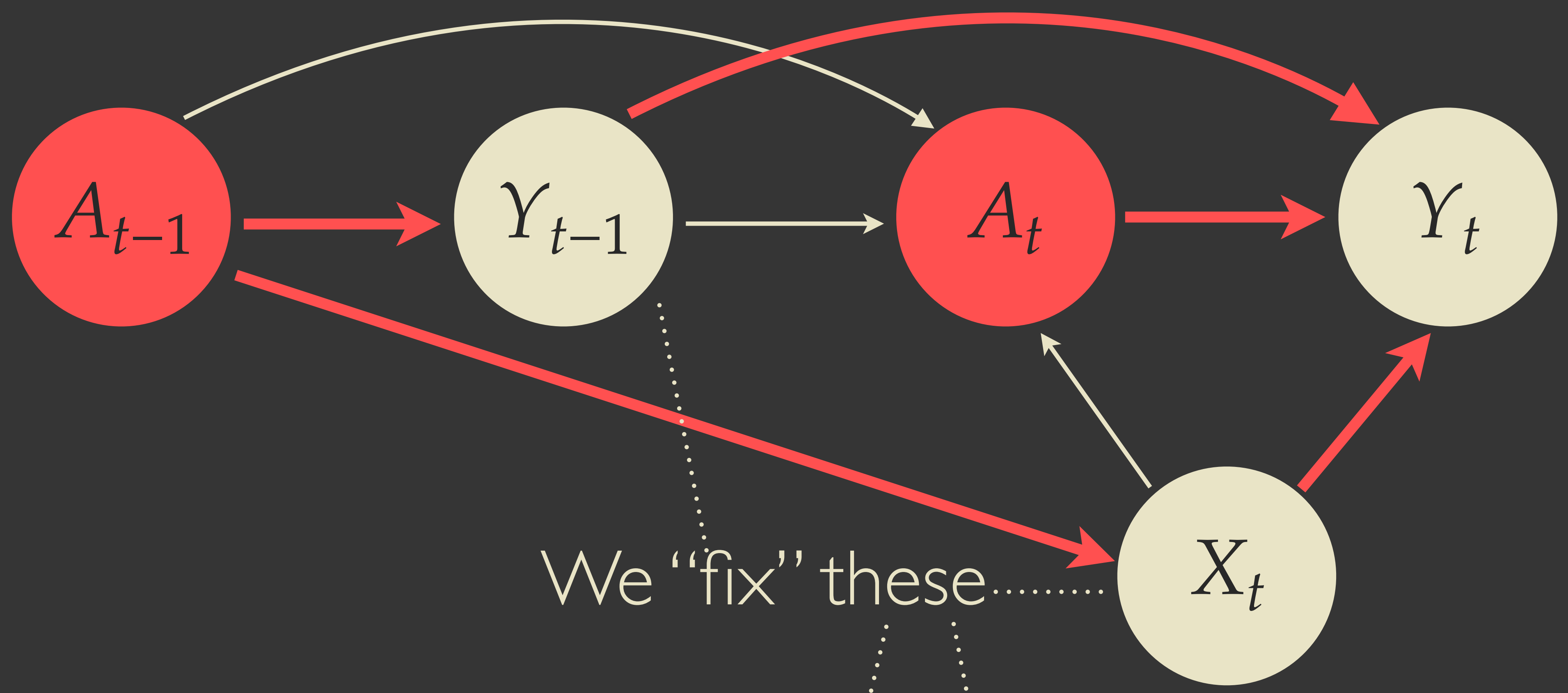

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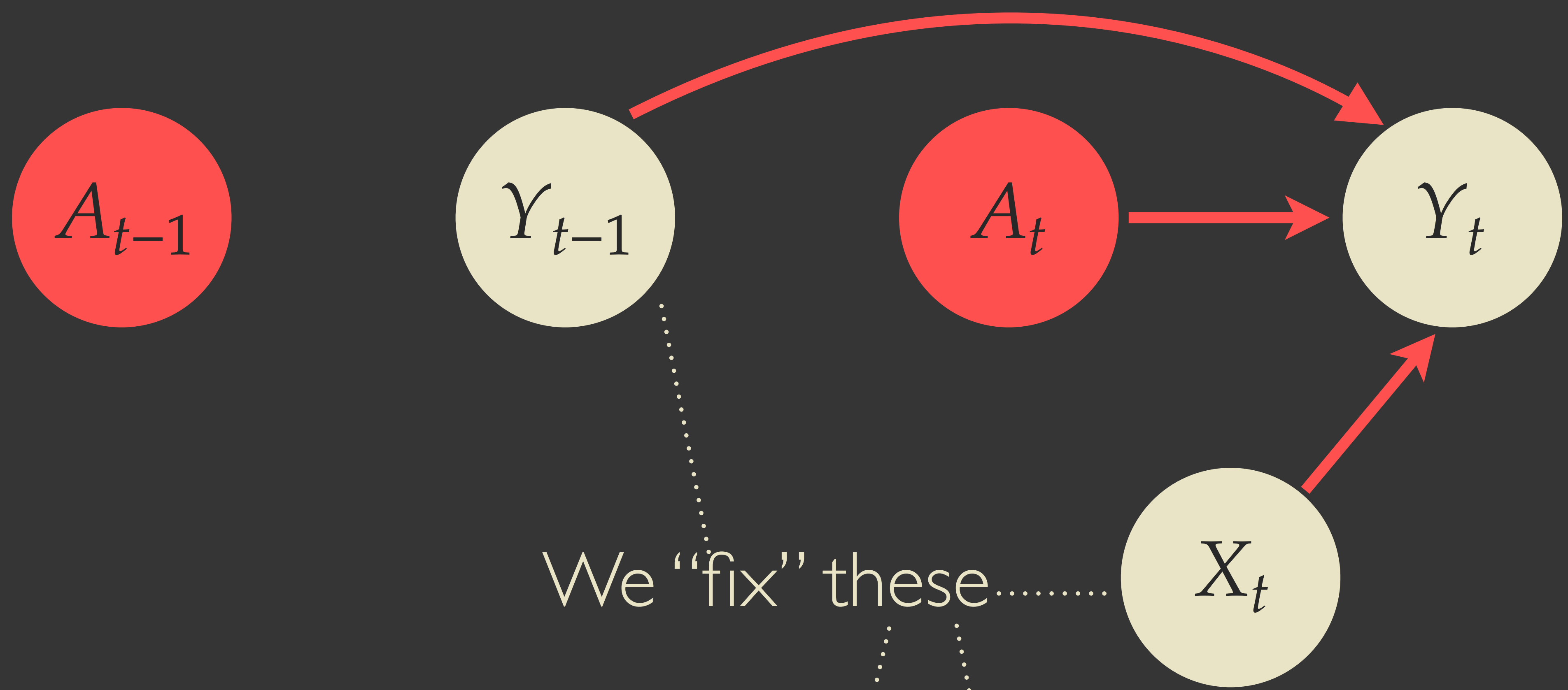

We “fix” these

$$Y_t = \beta_0 + \beta_1 A_t + \beta_2 X_t + \beta_3 Y_{t-1} + \beta_4 A_{t-1}$$

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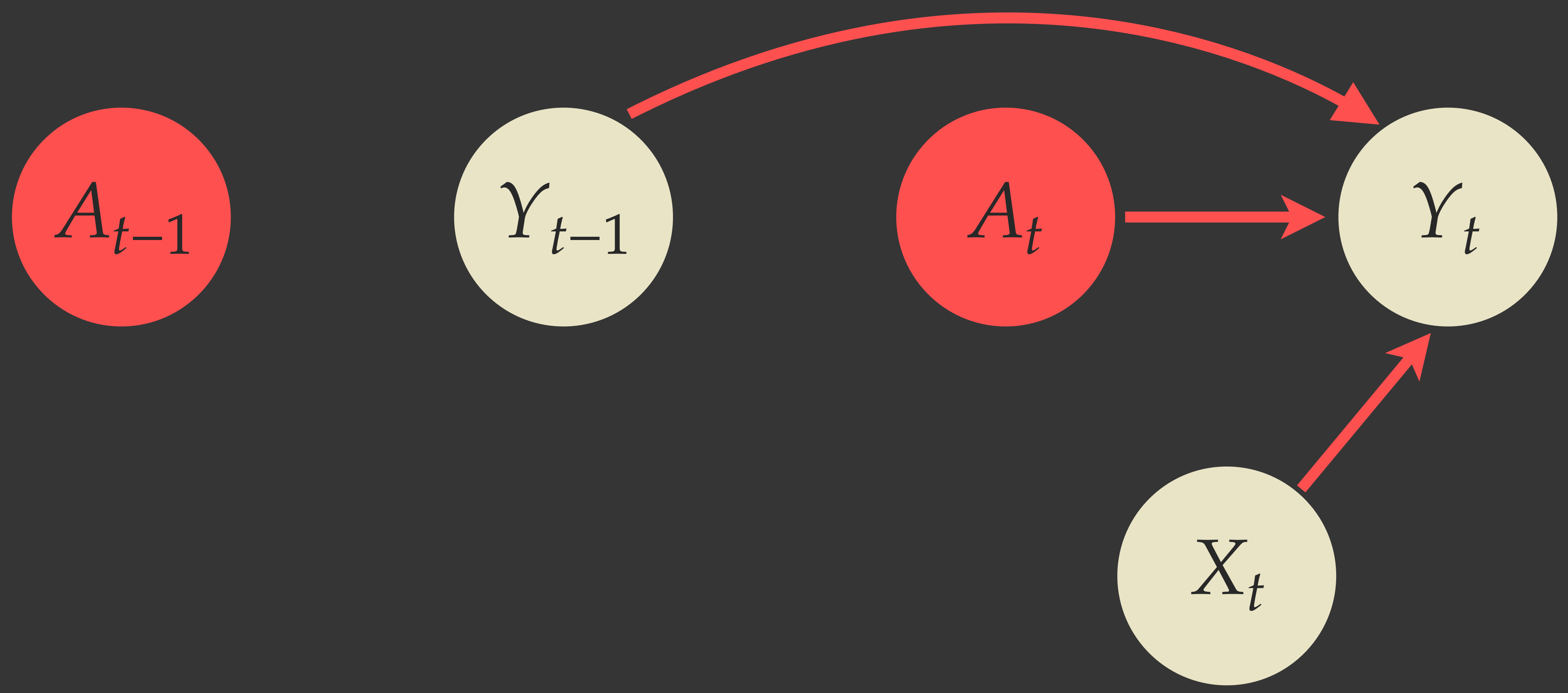

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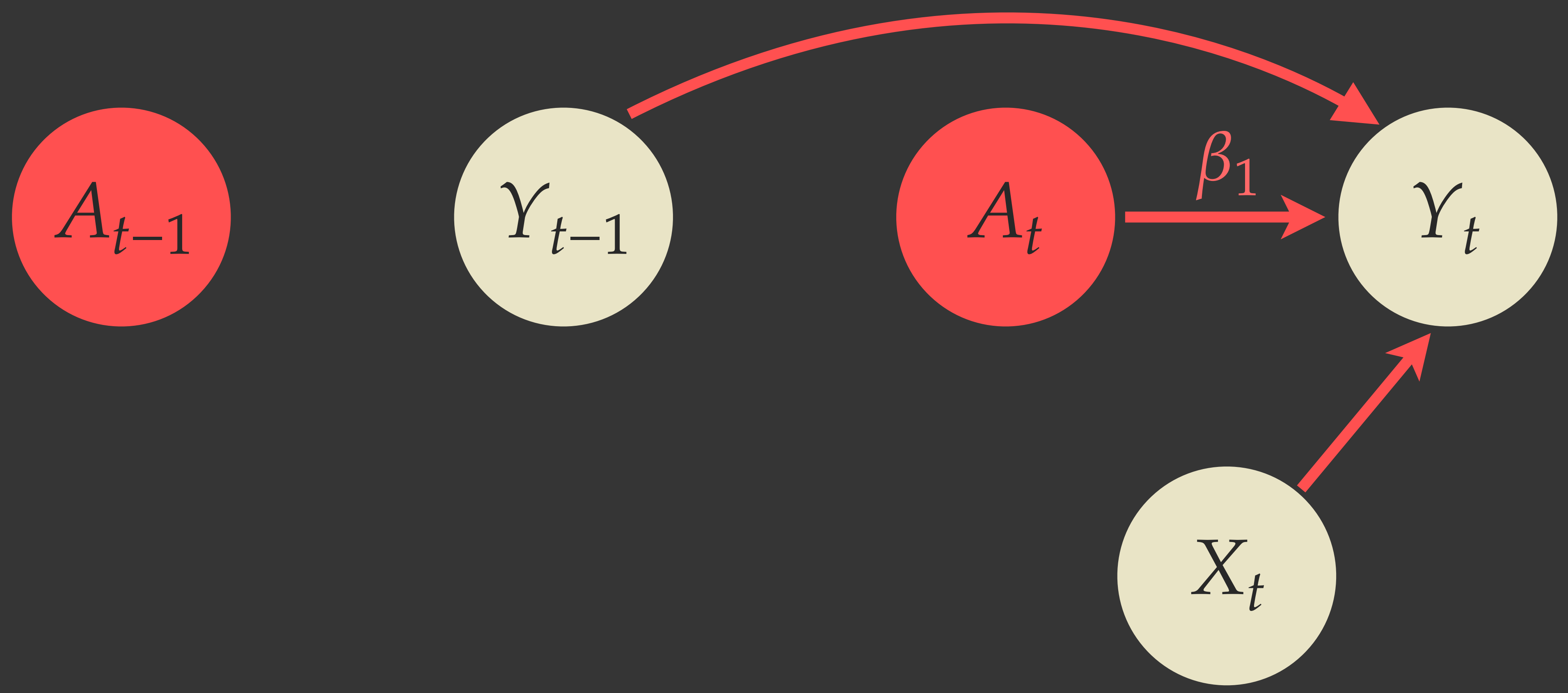

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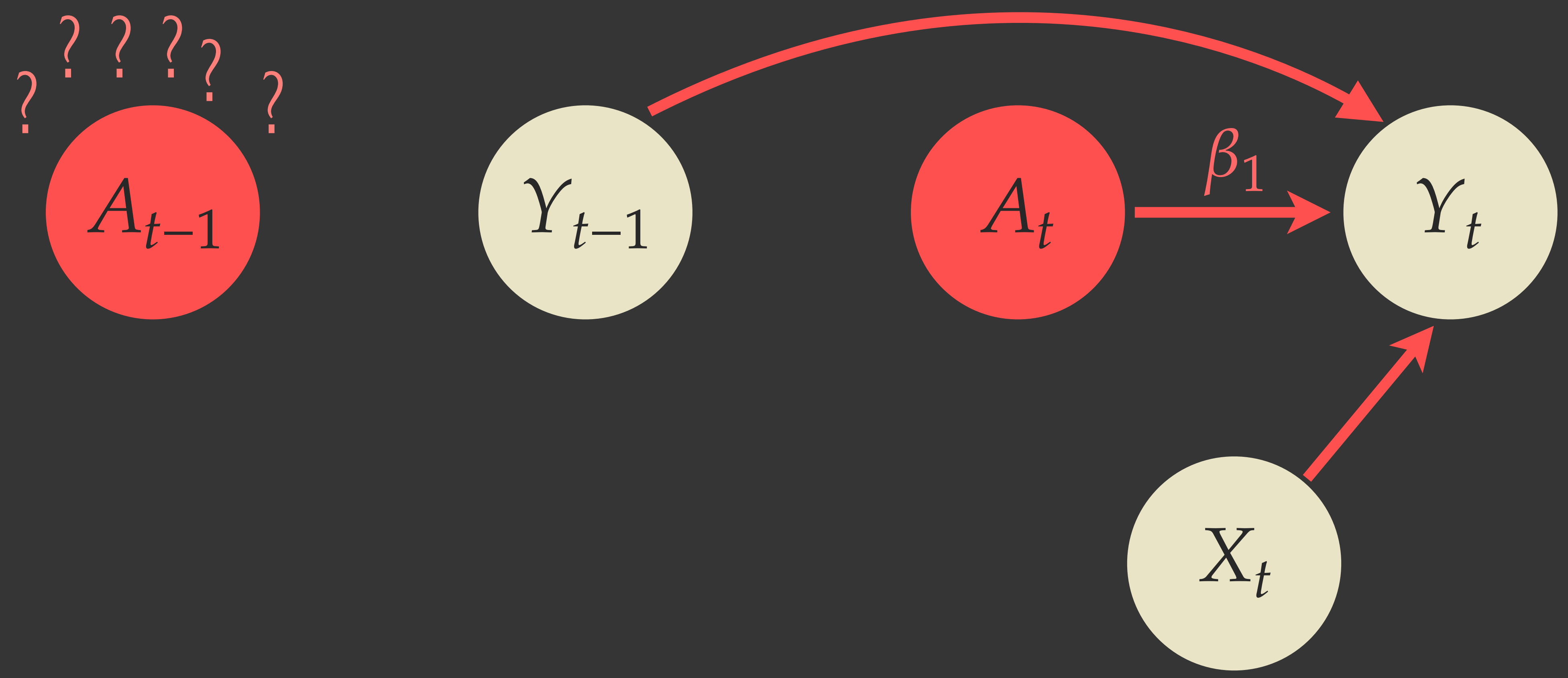

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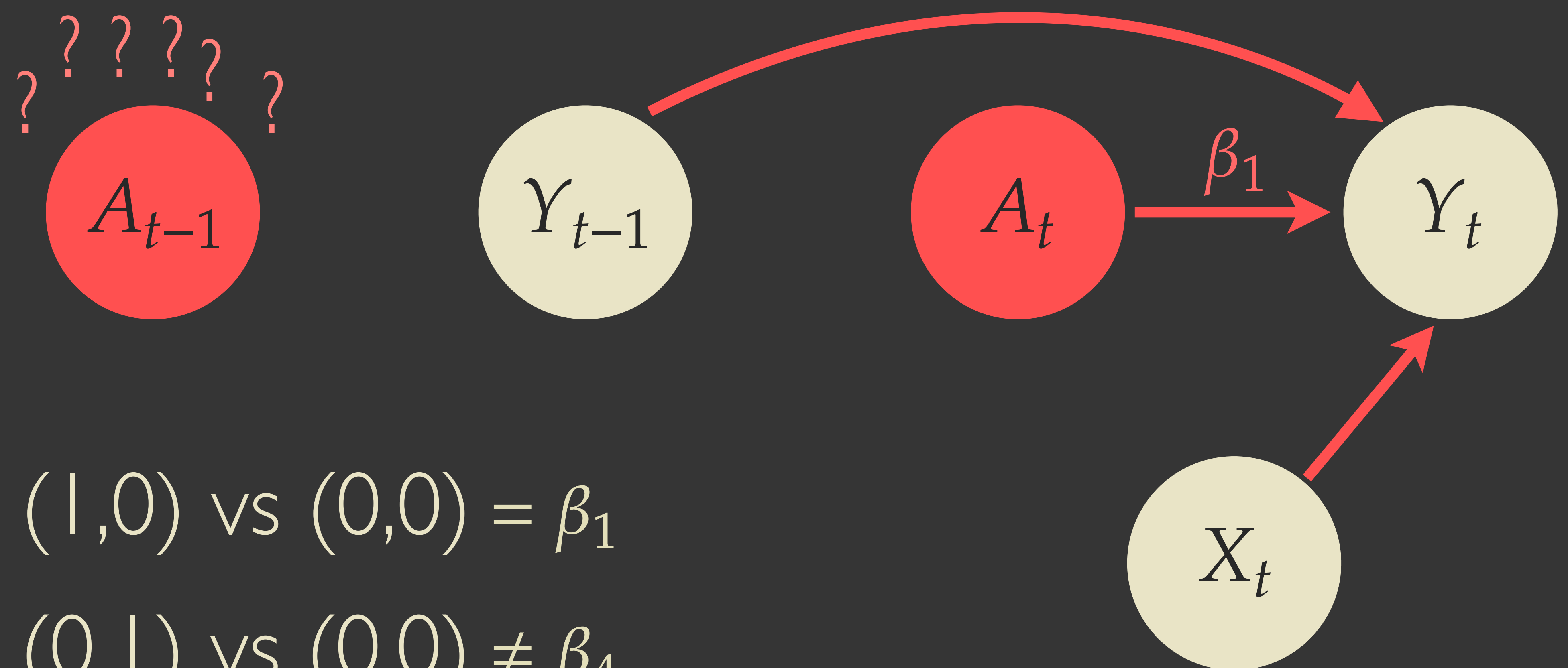

CET: (1,0) vs (0,0) $= \beta_1$

ATHE: (0,1) vs (0,0) $\neq \beta_4$

ATHE: (1,1) vs (0,0) $\neq \beta_1 + \beta_4$

$$Y_t = \beta_0 + \beta_1 A_t + \beta_2 X_t + \beta_3 Y_{t-1} + \beta_4 A_{t-1}$$

How weighting can help

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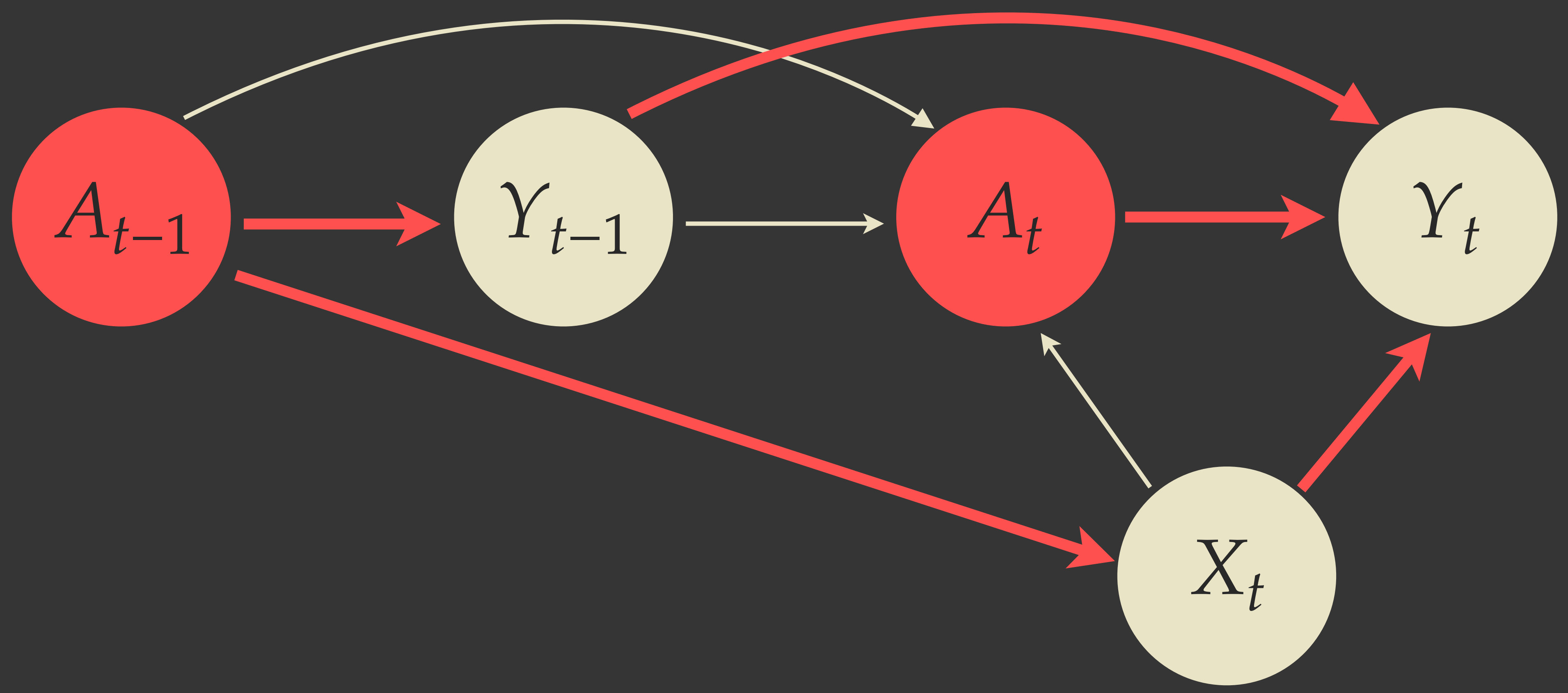

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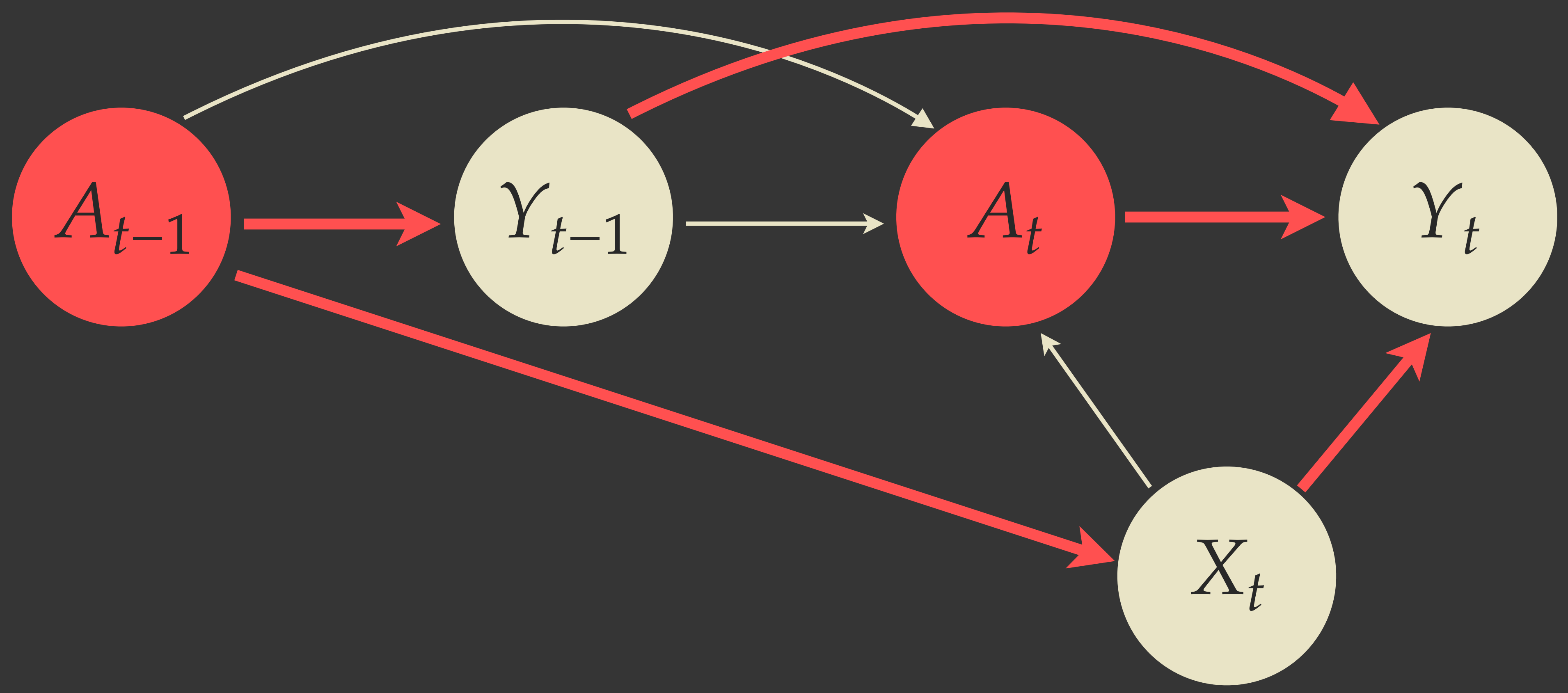
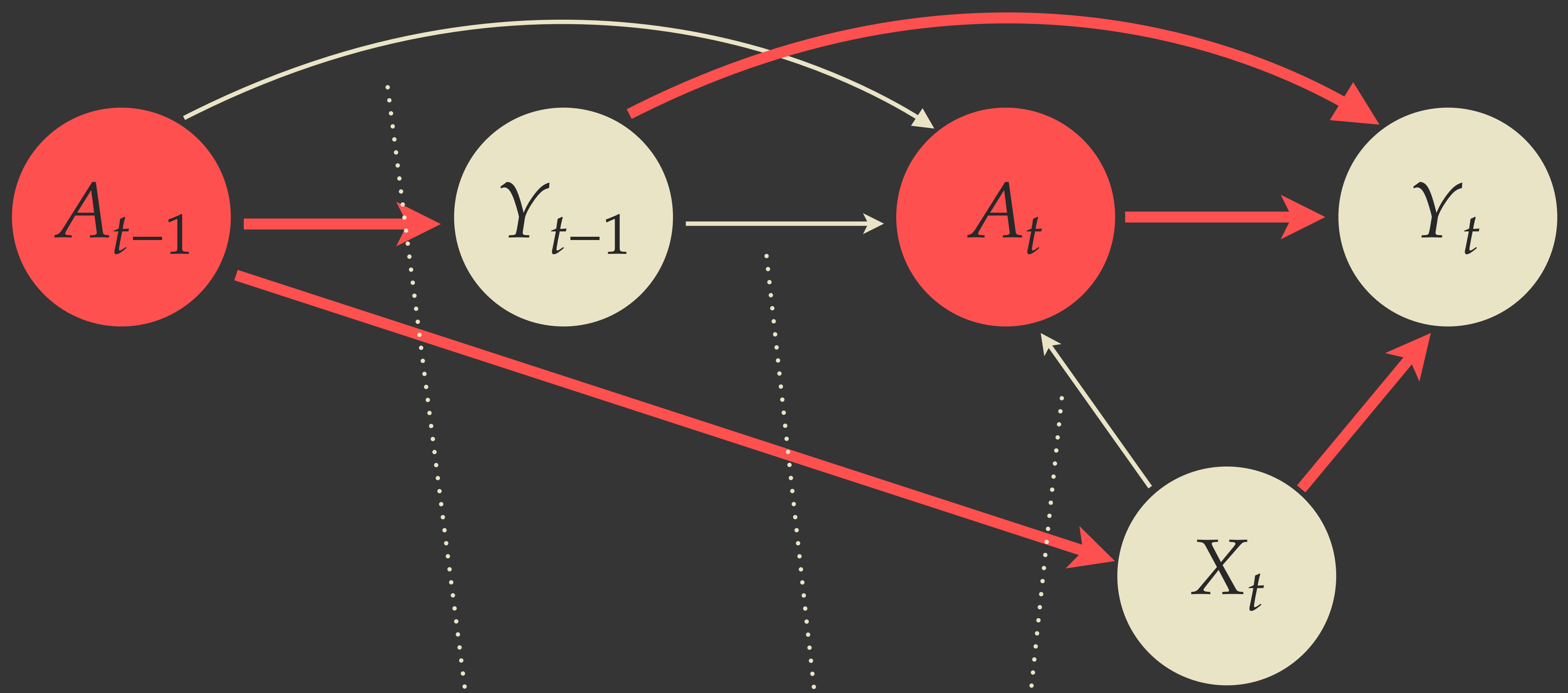


$$W_{it} = \prod_{s=1}^{t} \frac{1}{\Pr[A_{is}|A_{t-1}, X_t, Y_{t-1}]}.$$

How weighting can help



We weight to create balance

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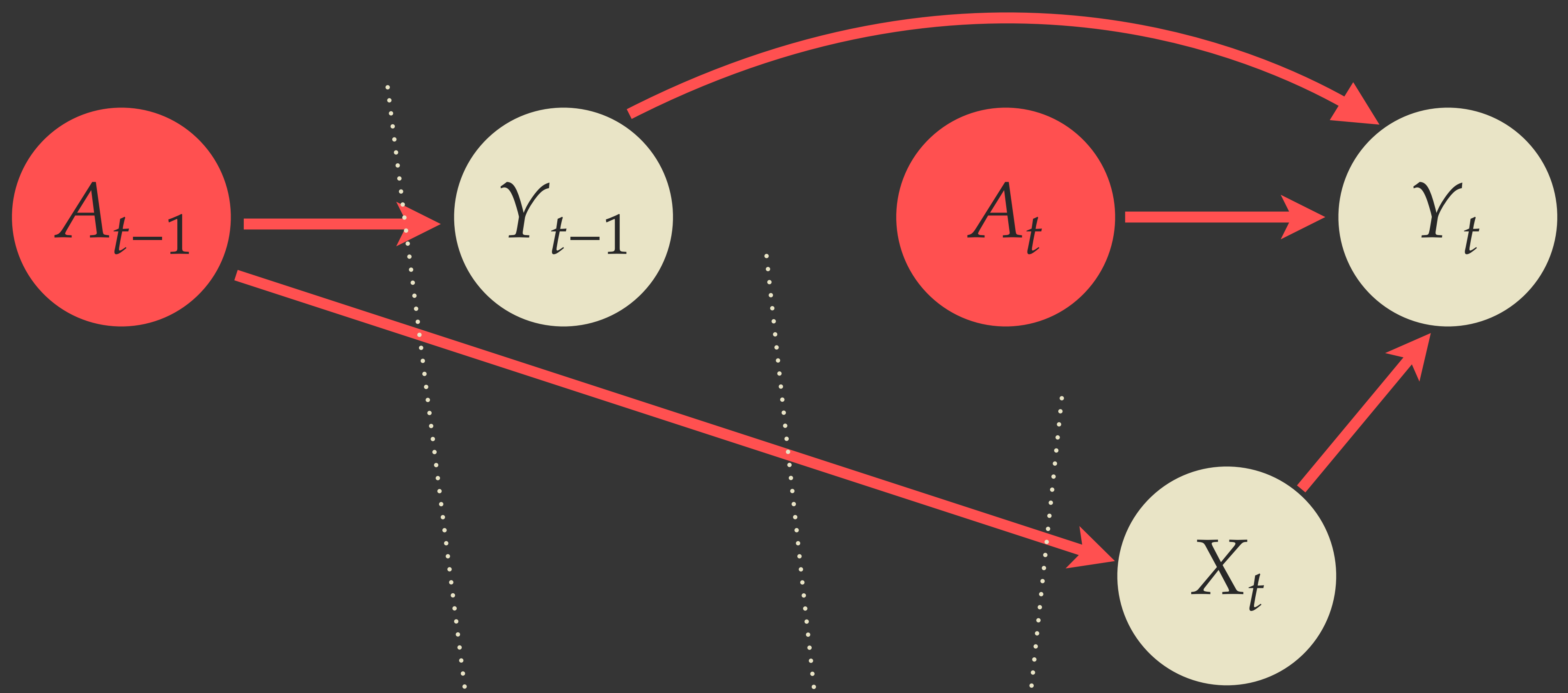

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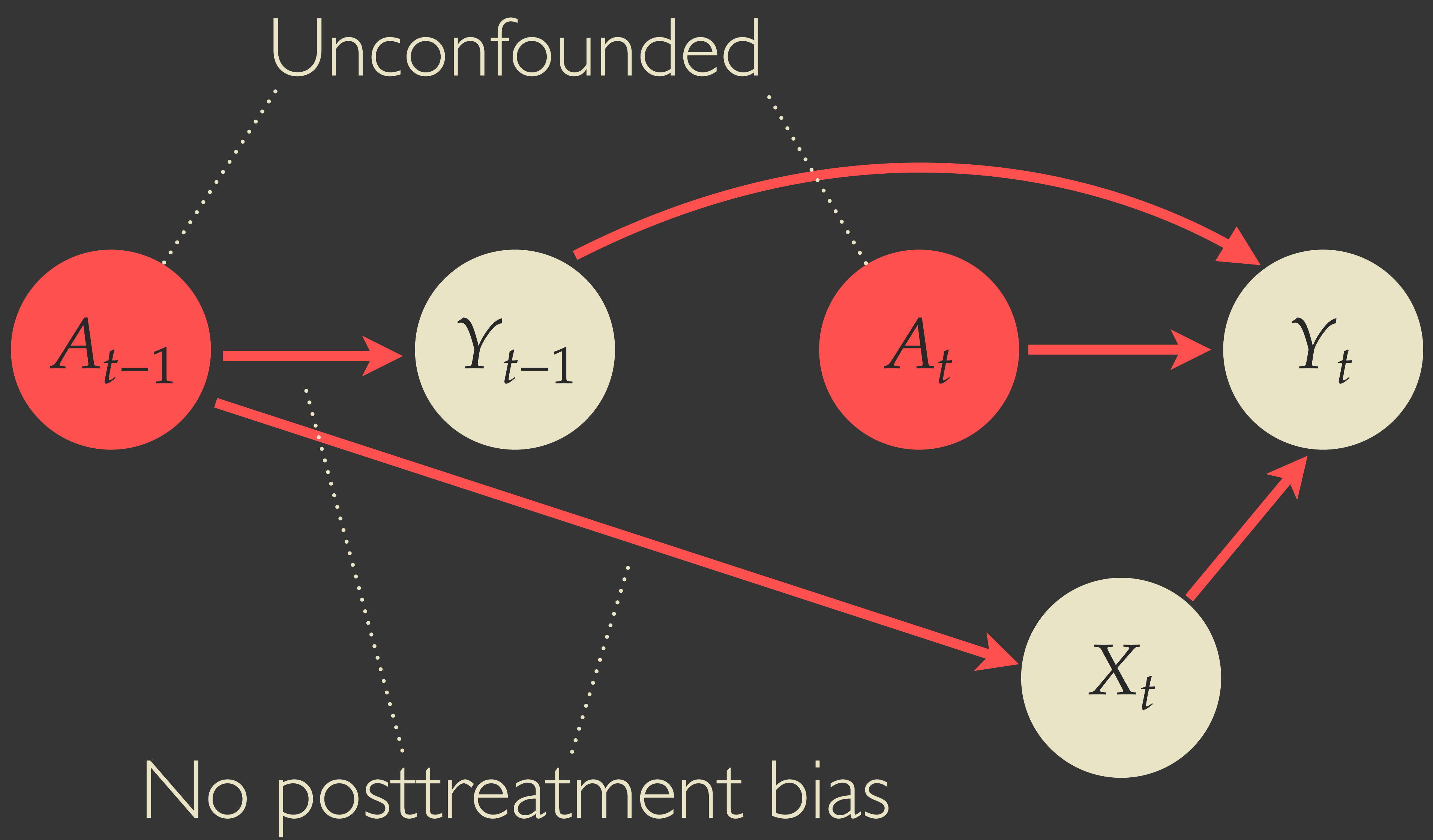

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How weighting can help

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$$
\begin{aligned}
E[Y_t(a_t, a_{t-1})] &= E_W[Y_t | A_t = a_t, A_{t-1} = a_{t-1}] \\
&= \beta_0 + \beta_1 a_t + \beta_2 a_{t-1}
\end{aligned}
$$

How weighting can help

WLS

$$E[Y_t(a_t, a_{t-1})] = E_W[Y_t | A_t = a_t, A_{t-1} = a_{t-1}]$$
$$= \beta_0 + \beta_1 a_t + \beta_2 a_{t-1}$$

How weighting can help

WLS

$$E[Y_t(a_t, a_{t-1})] = E_W[Y_t | A_t = a_t, A_{t-1} = a_{t-1}]$$
$$= \beta_0 + \beta_1 a_t + \beta_2 a_{t-1}$$

CET: (1,0) vs (0,0) $= \beta_1$

ATHE: (0,1) vs (0,0) $= \beta_2$

ATHE: (1,1) vs (0,0) $= \beta_1 + \beta_2$

The Long Arm of the Democratic Peace?

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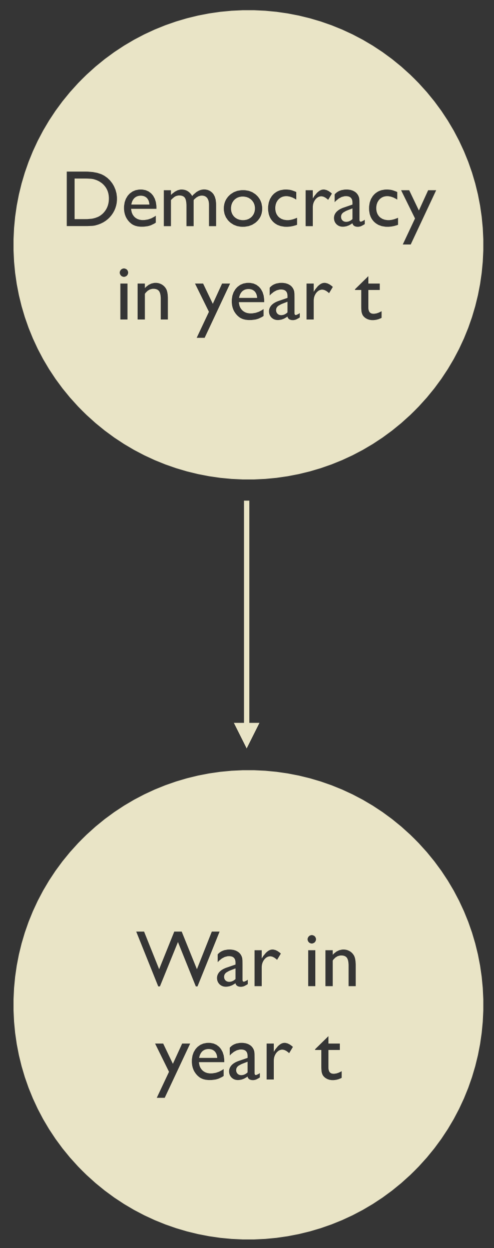

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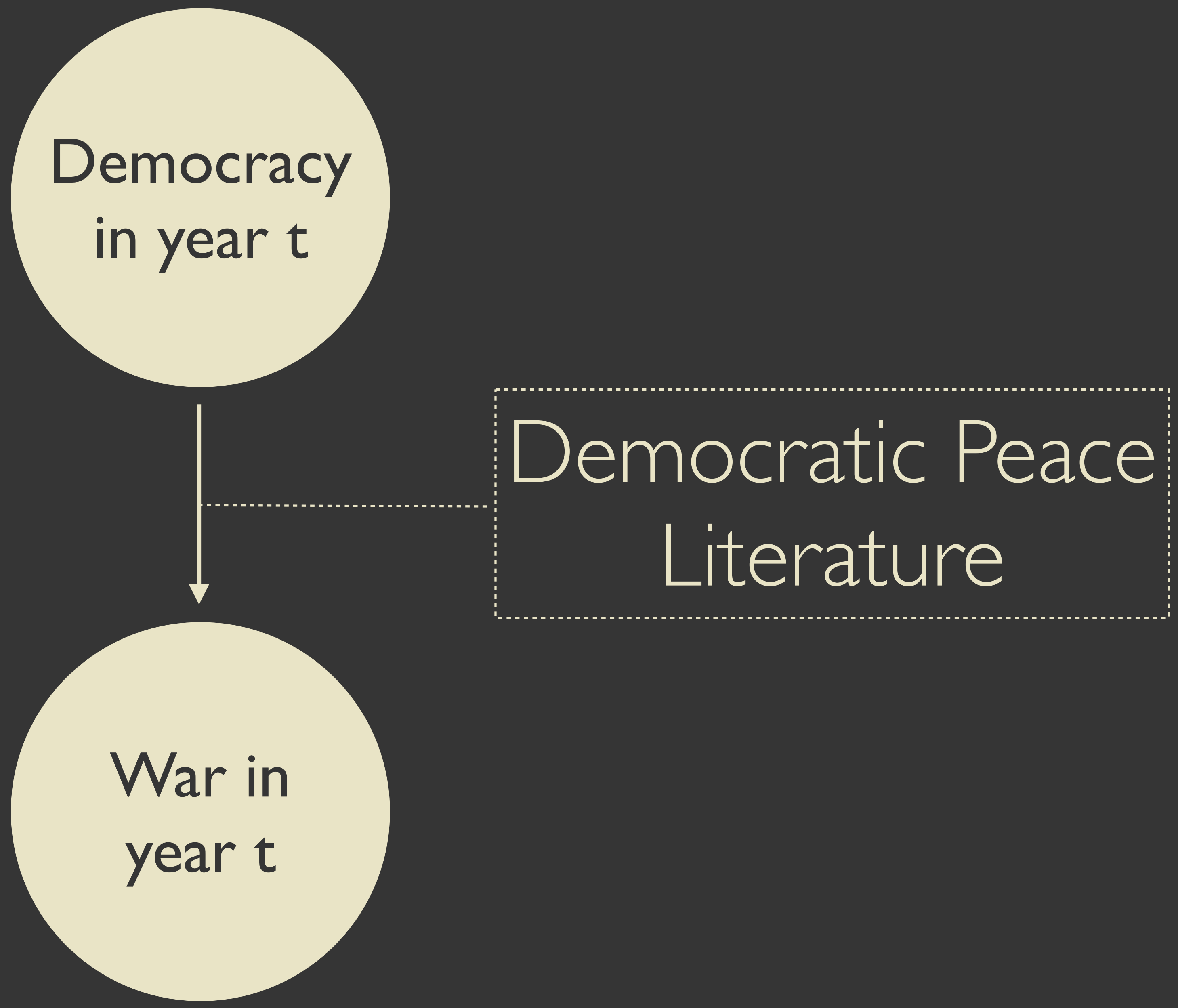

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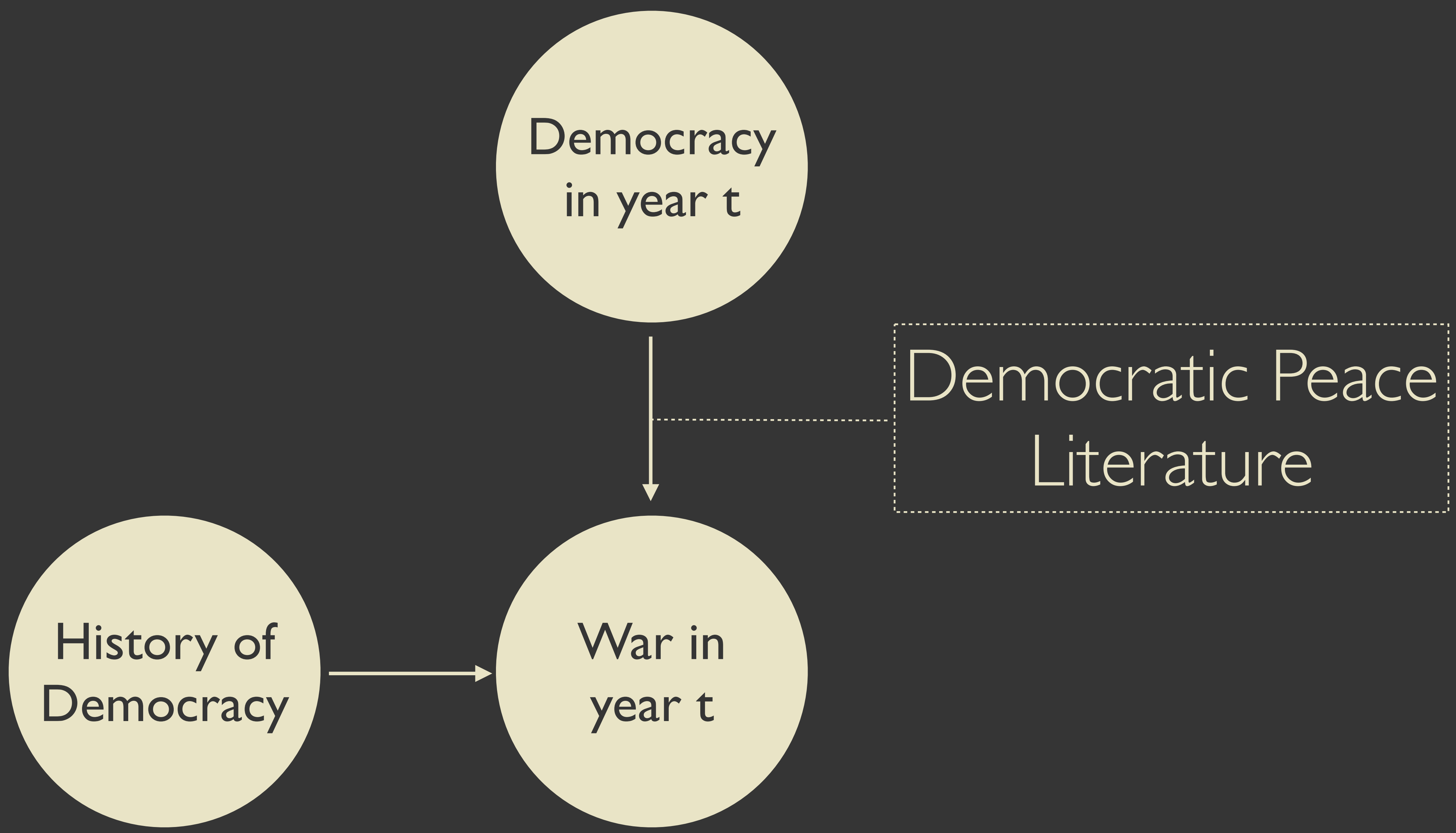

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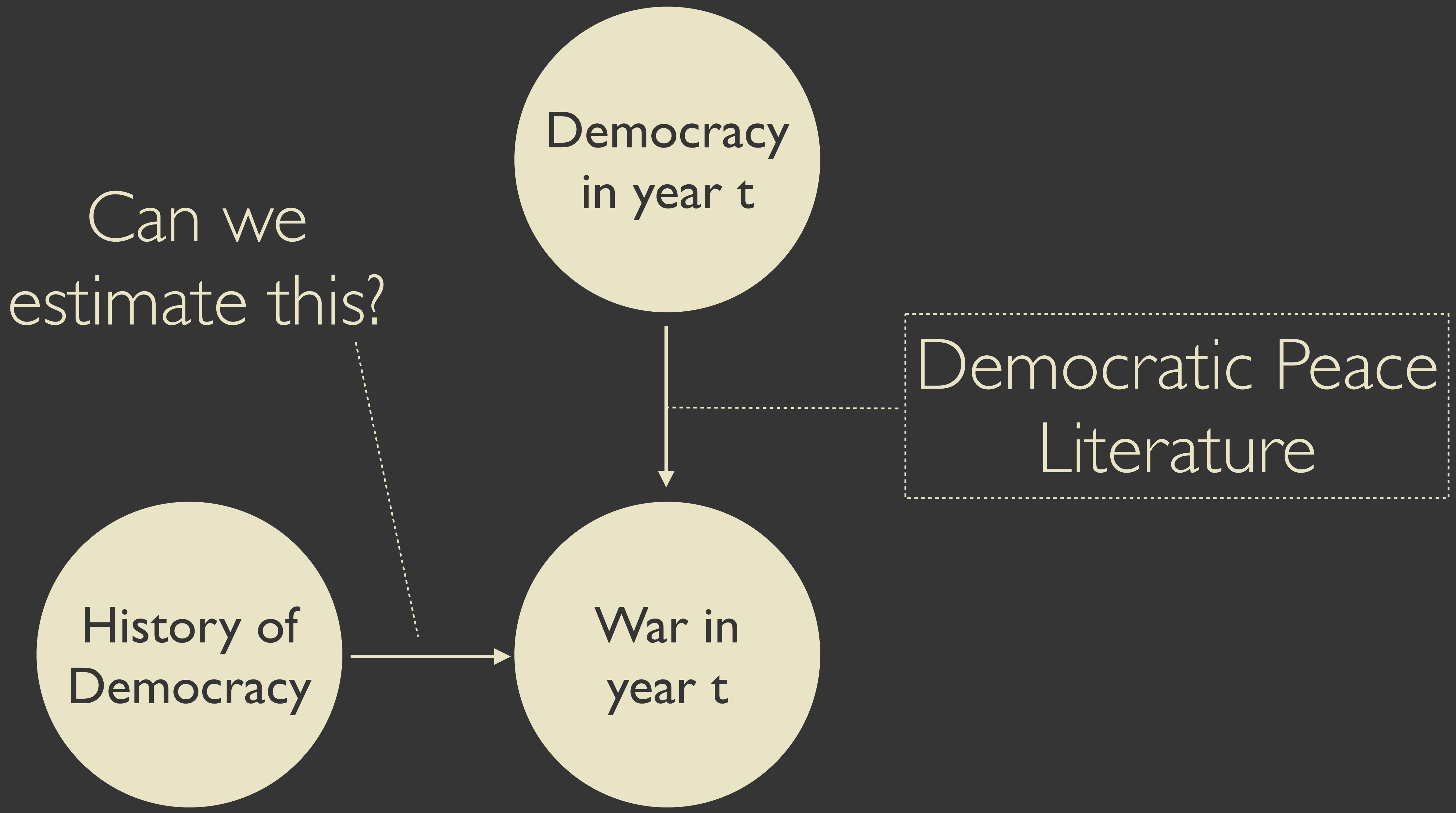


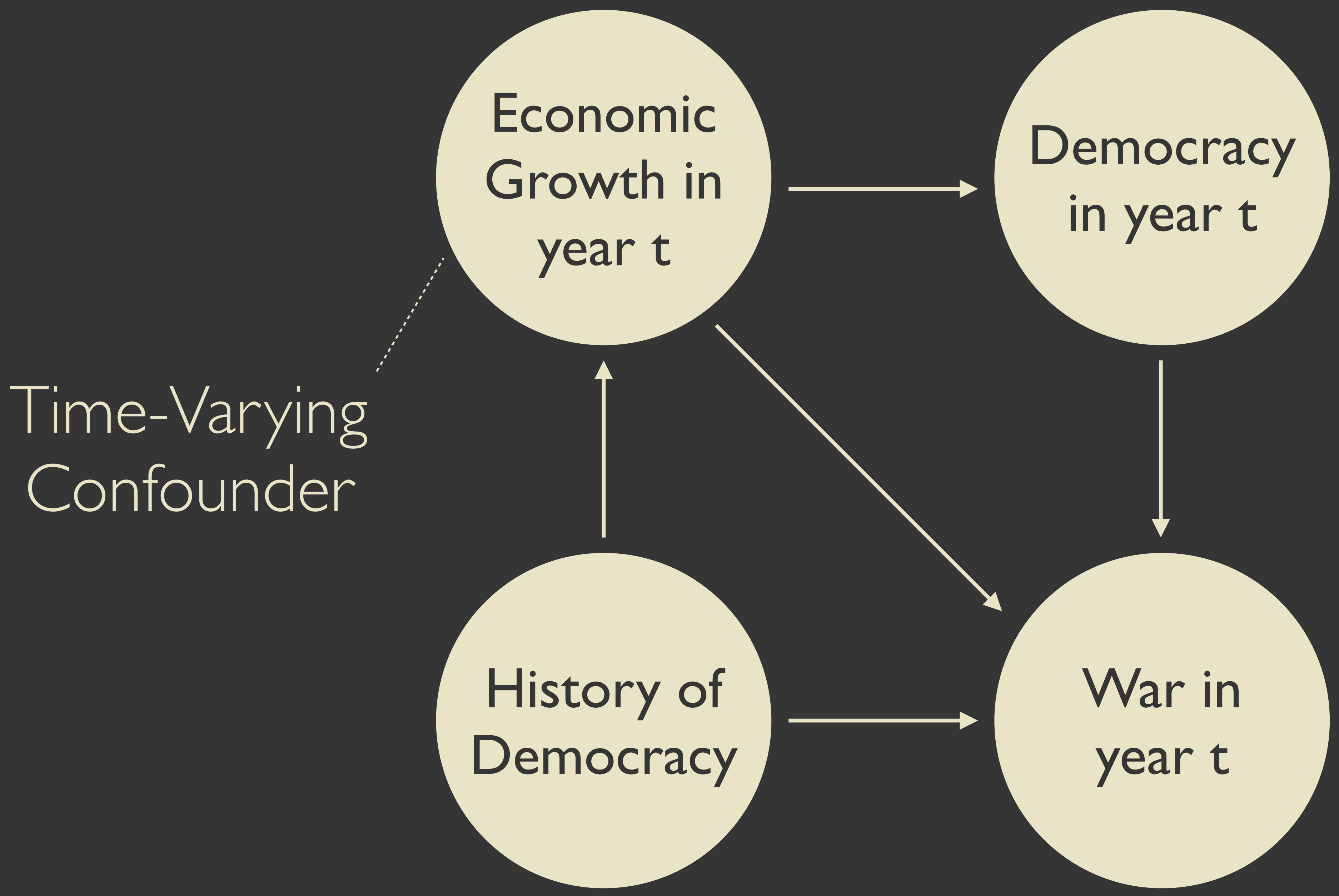
Revisiting Beck, Katz, and Tucker (1998)

	Dependent variable: Dispute
	BKT Model
	(1)
Democracy Blip	-0.651^{***}
	(0.160)
Cumulative Democracy	
Growth	-3.837^{***}
Observations	20,448
Note:	*p<0.1; **p<0.05; ***p<0.01

Revisiting Beck, Katz, and Tucker (1998)

	Dependent variable: Dispute	
	BKT Model	Misspecified Cumulative Model
	(1)	(2)
Democracy Blip	-0.651^{***}	
	(0.160)	
Cumulative Democracy		-0.010
		(0.012)
Growth	-3.837^{***}	-4.360^{***}
Observations	20,448	20,448
Note:		*p<0.1; **p<0.05; ***p<0.01

Misspecification of an ATHE



Revisiting Beck, Katz, and Tucker (1998)

	Dependent variable: Dispute		
	BKT Model	Misspecified Cumulative Model	IPTW + MSM
	(1)	(2)	(3)
Democracy Blip	-0.651^{***}		
	(0.160)		
Cumulative Democracy		-0.010	-0.045^{***}
		(0.012)	(0.013)
Growth	-3.837^{***}	-4.360^{***}	
Observations	20,448	20,448	20,448
Note:		*p<0.1; **p<0.05; ***p<0.01	

TSCS data under unmeasured confounding

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$$Y_{it}(\underline{a}_t) \perp\!\!\!\perp A_{it} | \underline{X}_{it}, \underline{A}_{i,t-1} = \underline{a}_{t-1}, U$$

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$$Y_{it}(\underline{a}_t) \perp\!\!\!\perp A_{it} | \underline{X}_{it}, \underline{A}_{i,t-1} = \underline{a}_{t-1}, U$$

Treatment is unrelated
to the potential outcomes

TSCS data under unmeasured confounding

$$Y_{it}(\underline{a}_t) \perp\!\!\!\perp A_{it} | \underline{X}_{it}, \underline{A}_{i,t-1} = \underline{a}_{t-1}, U$$

Treatment is unrelated
to the potential outcomes

...conditional on the
covariate history

TSCS data under unmeasured confounding

...and a time-fixed
unmeasured confounder.

$$Y_{it}(\underline{a}_t) \perp\!\!\!\perp A_{it} | \underline{X}_{it}, \underline{A}_{i,t-1} = \underline{a}_{t-1}, U$$

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How unit-specific weighting can help

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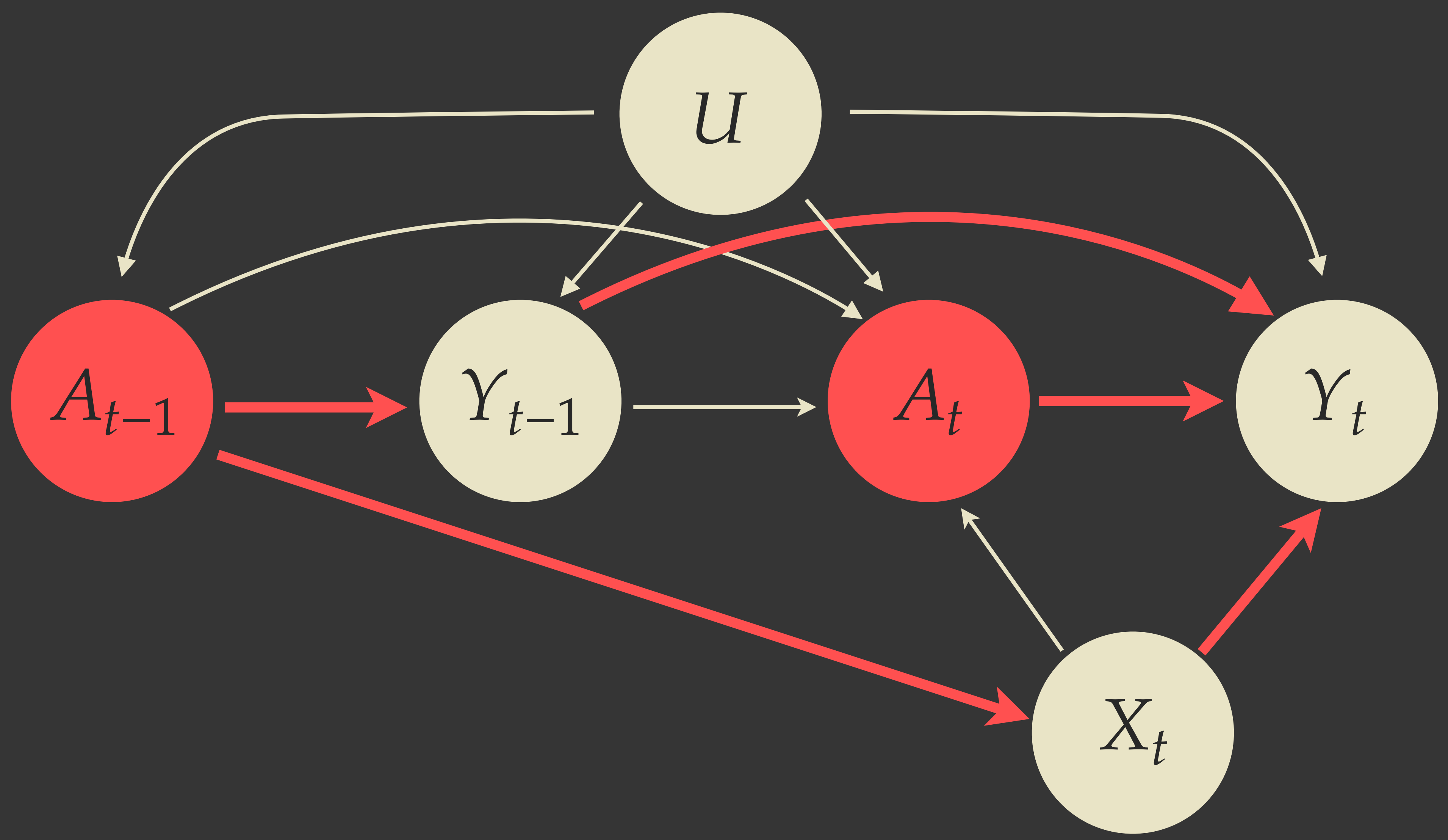
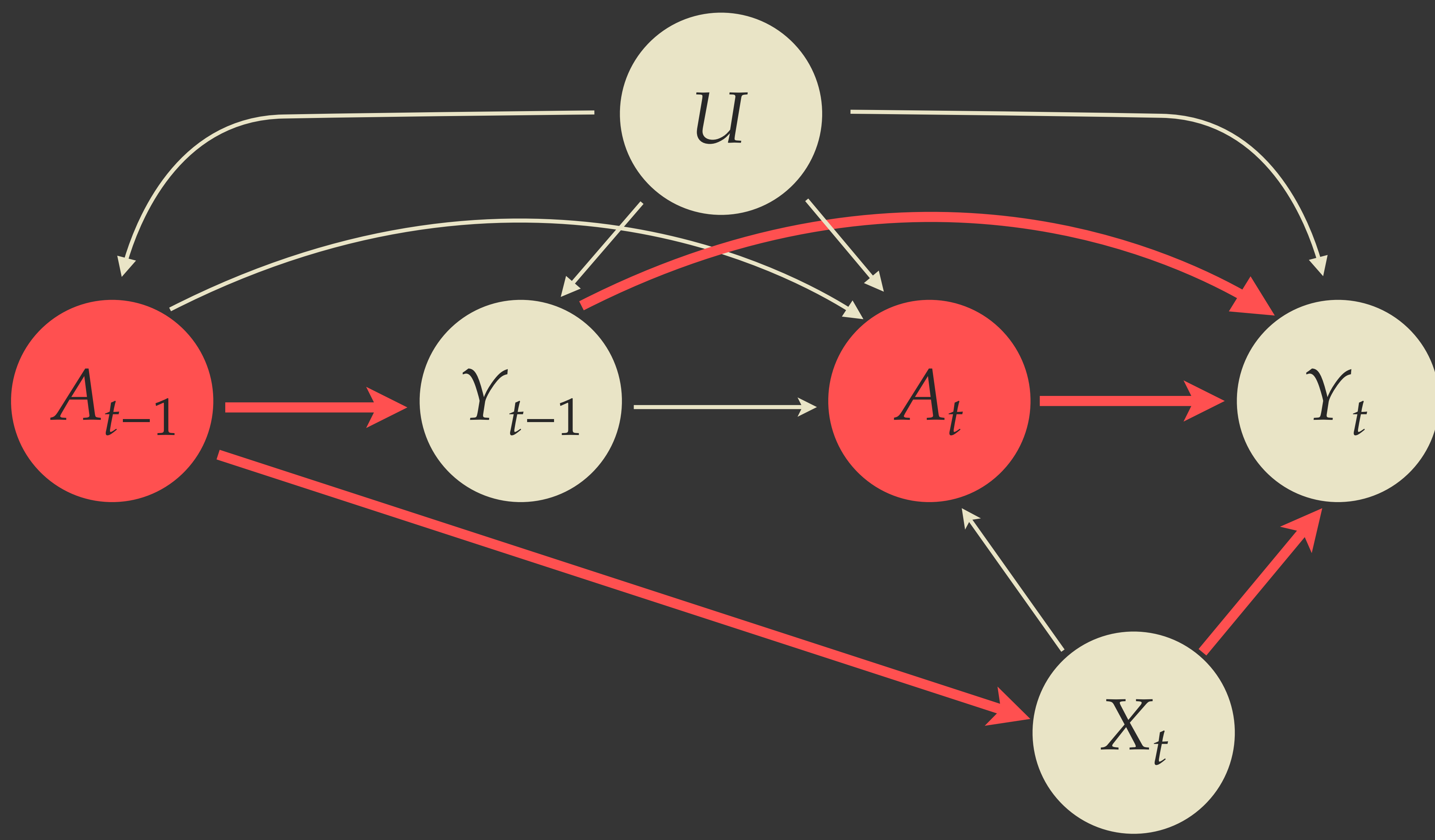


How unit-specific weighting can help



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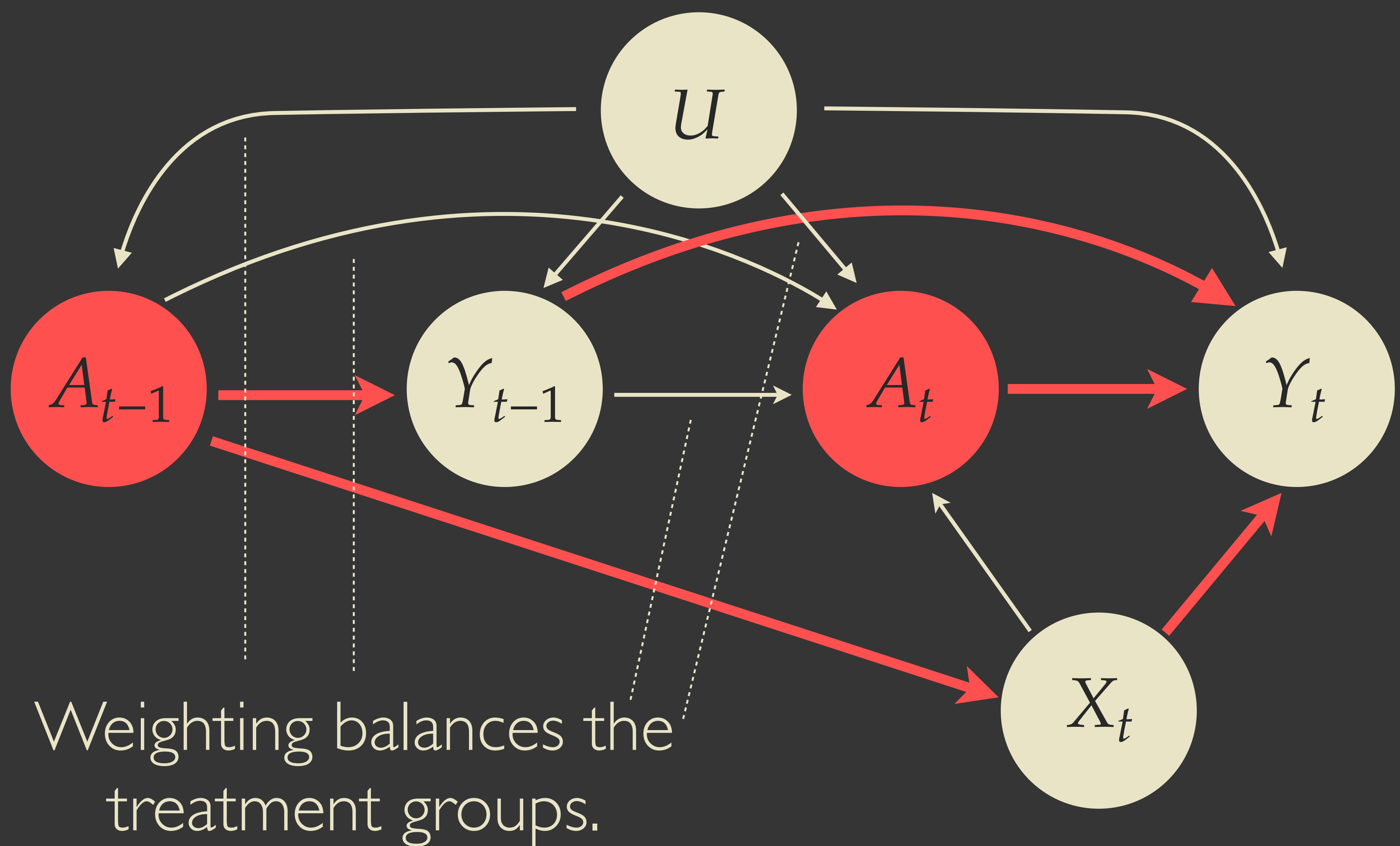
How unit-specific weighting can help



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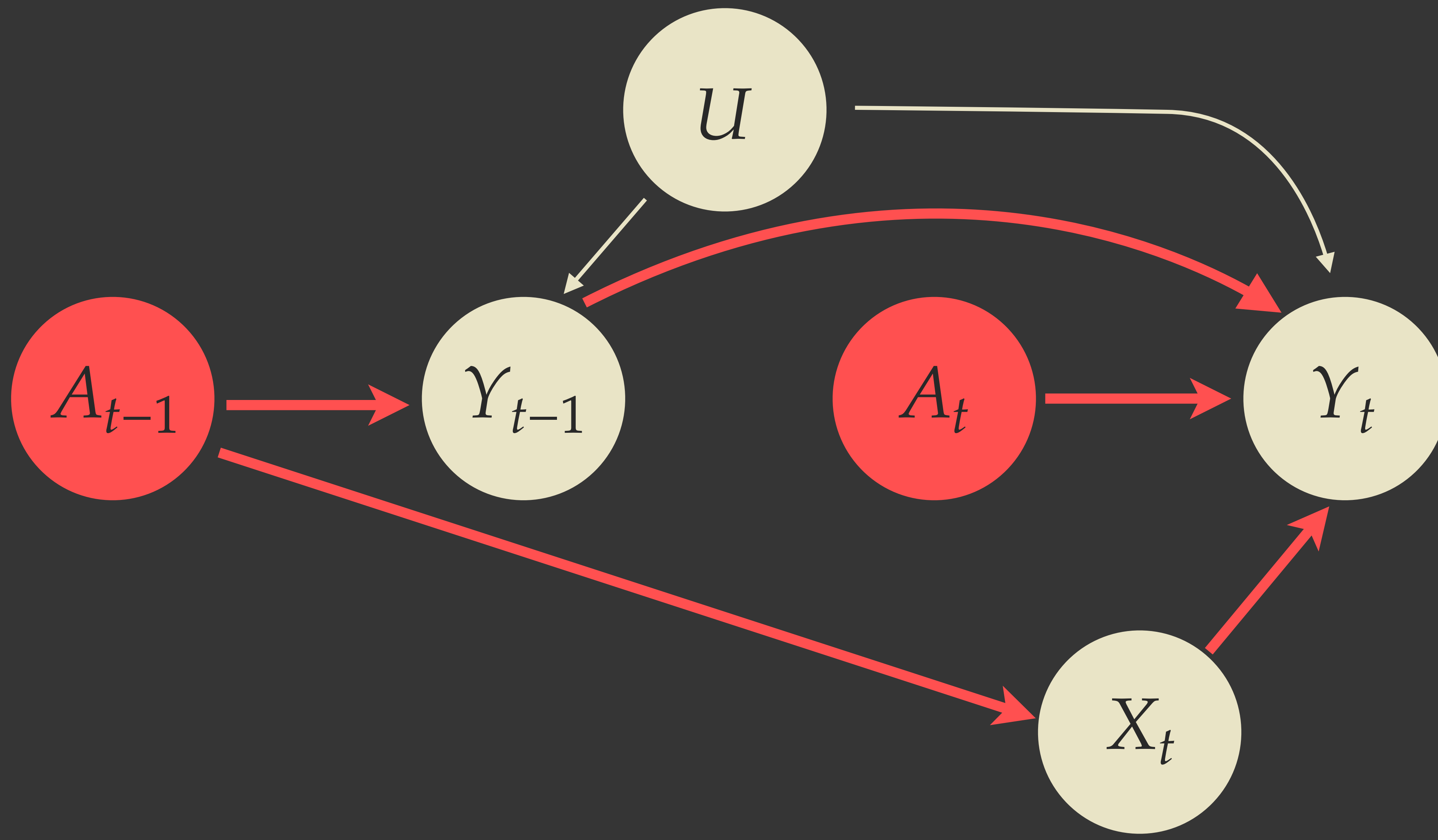

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A weighting approach to fixed effects

A weighting approach to fixed effects

1. Estimate unit-specific probability of treatment over time and construct weights.

A weighting approach to fixed effects

1. Estimate unit-specific probability of treatment over time and construct weights.
2. Estimate a pooled outcome model with unit-specific weights

k-order sequential ignorability

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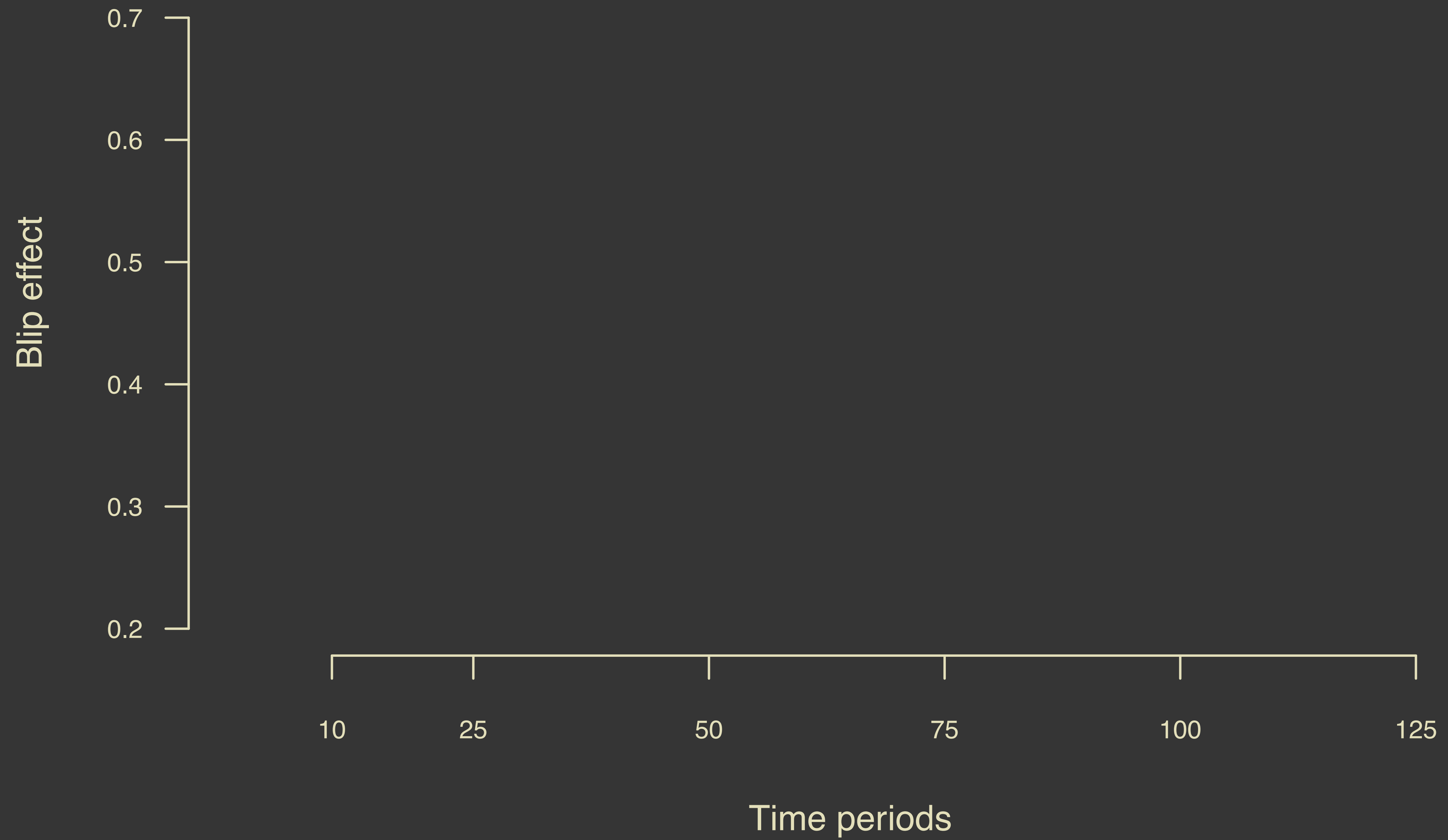
$$Y_{it}(\underline{a}_t) \perp\!\!\!\perp A_{it} | \underline{X}_{i,t:t-k}, \underline{A}_{i,t-1:t-k} = \underline{a}_{t-1:t-k}, U$$

k-order sequential ignorability

$$Y_{it}(\underline{a}_t) \perp\!\!\!\perp A_{it} | \underline{X}_{i,t:t-k}, \underline{A}_{i,t-1:t-k} = \underline{a}_{t-1:t-k}, U$$

Only the last k periods matter.

Blip effect: (1,0) vs (0,0)



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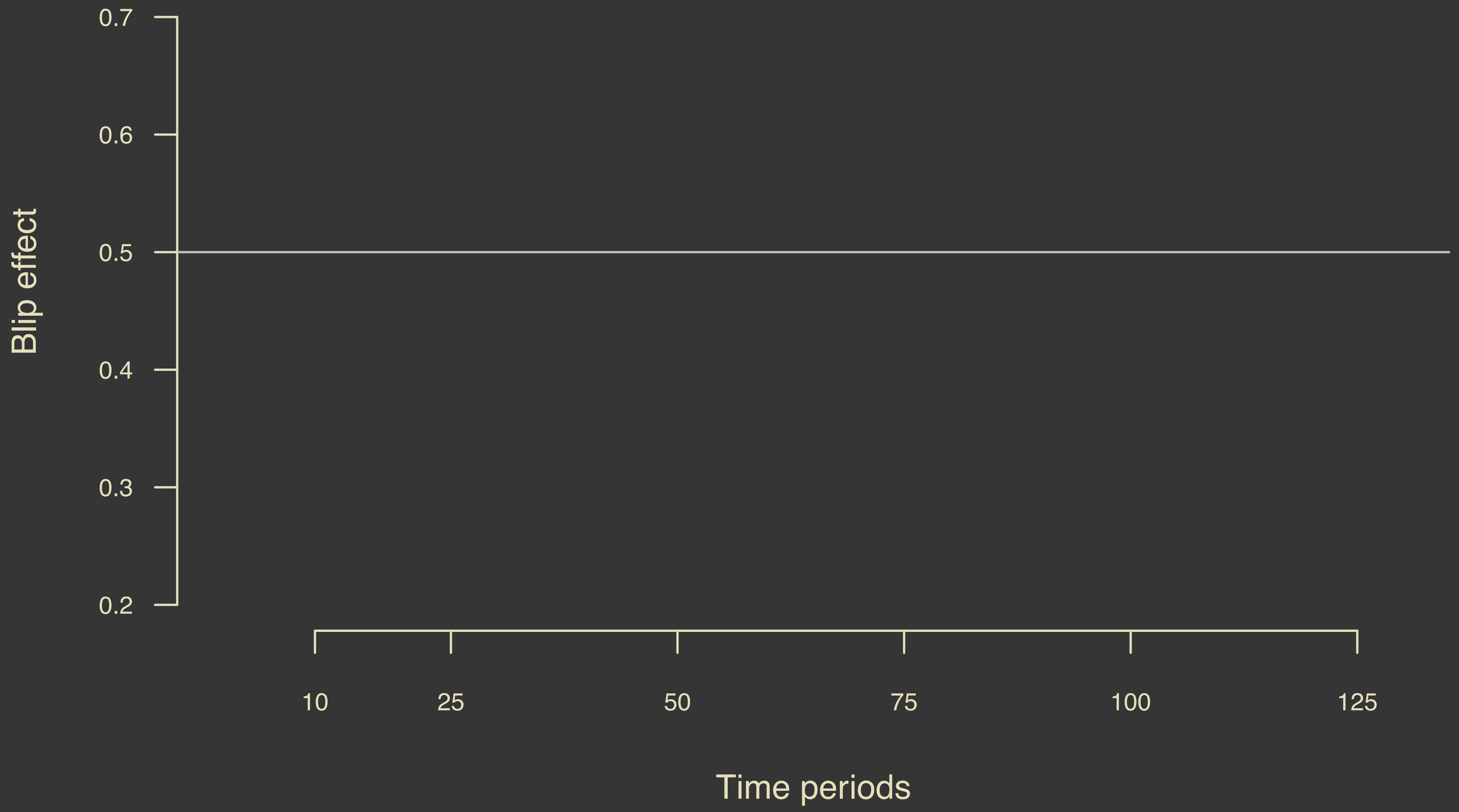

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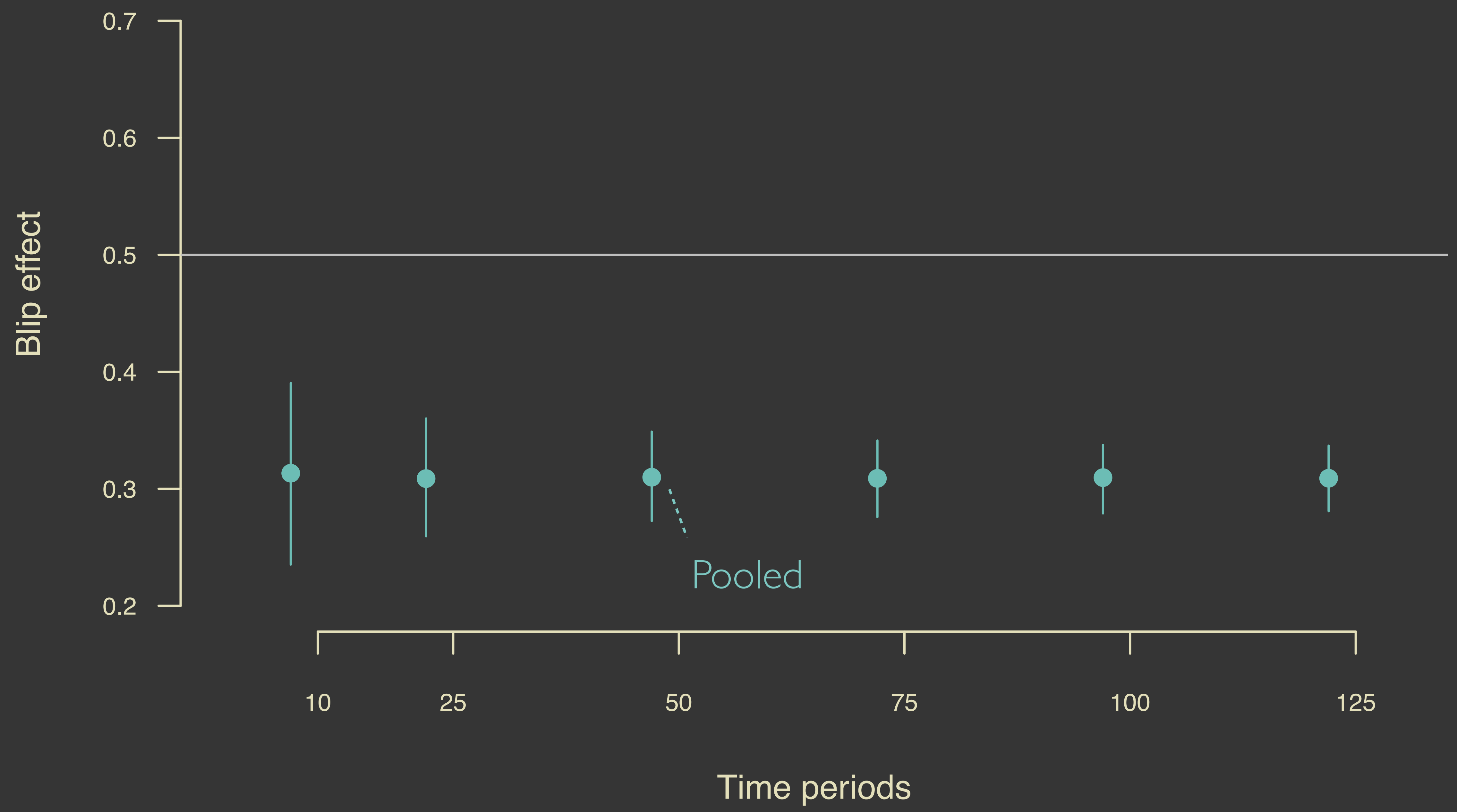

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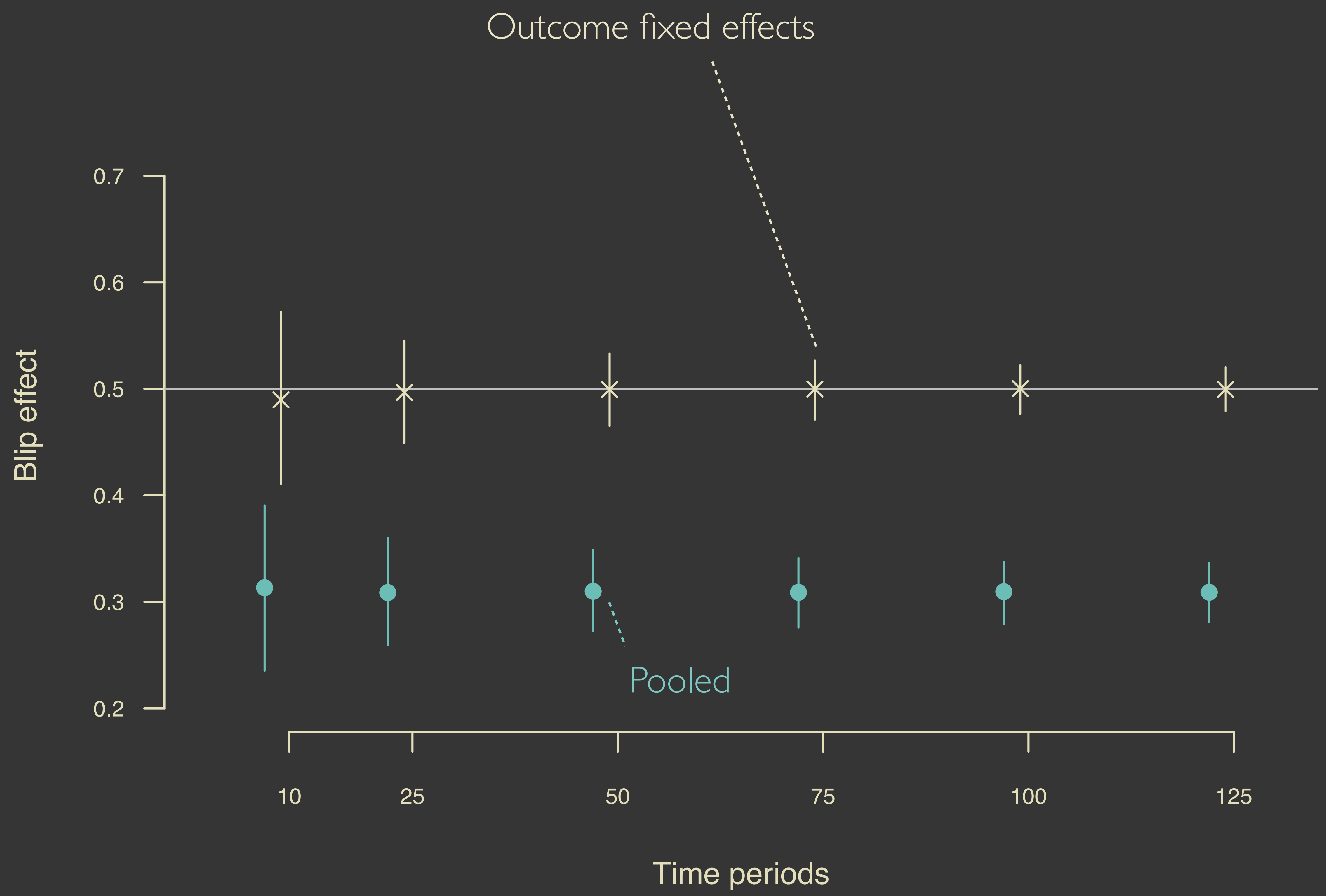

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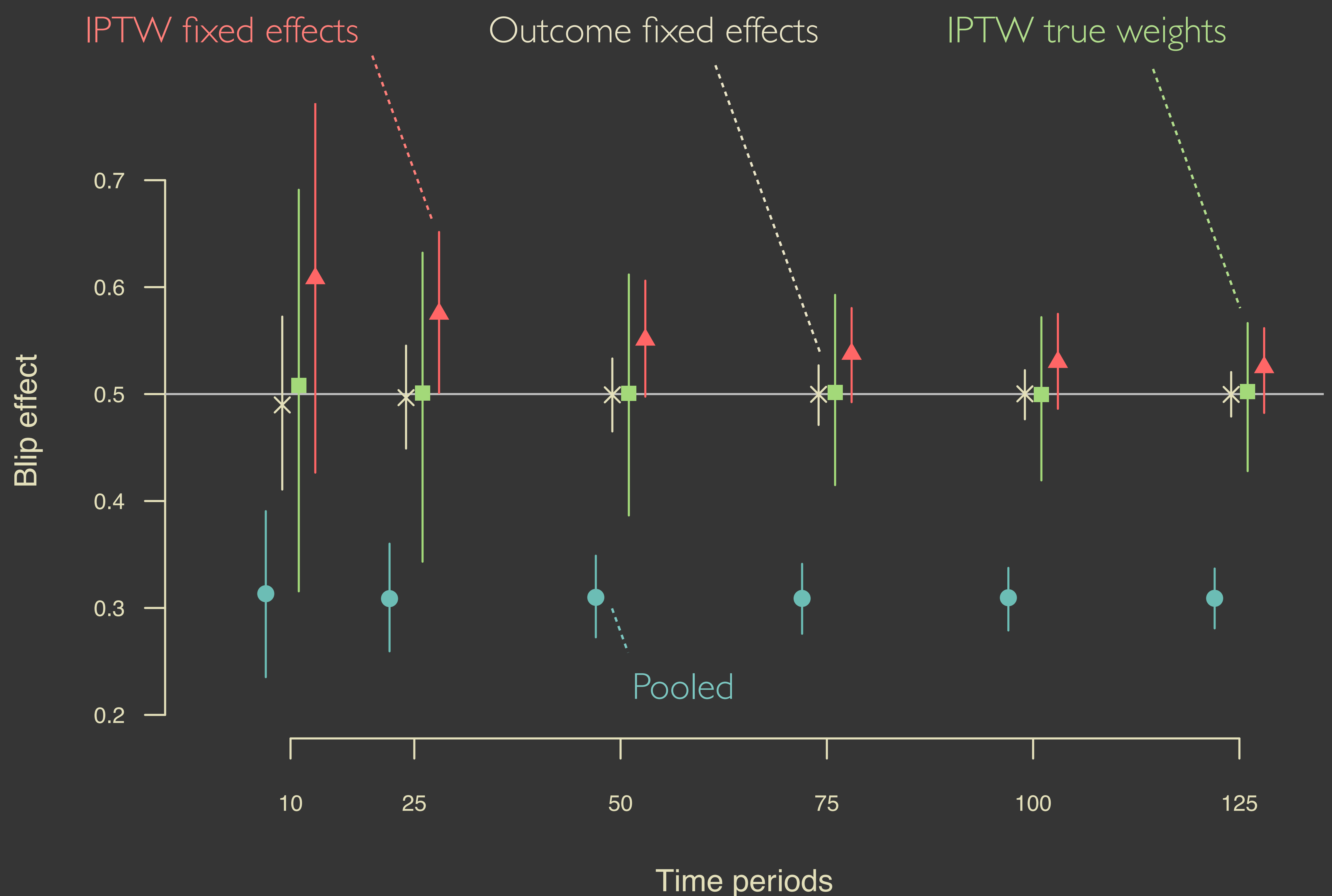
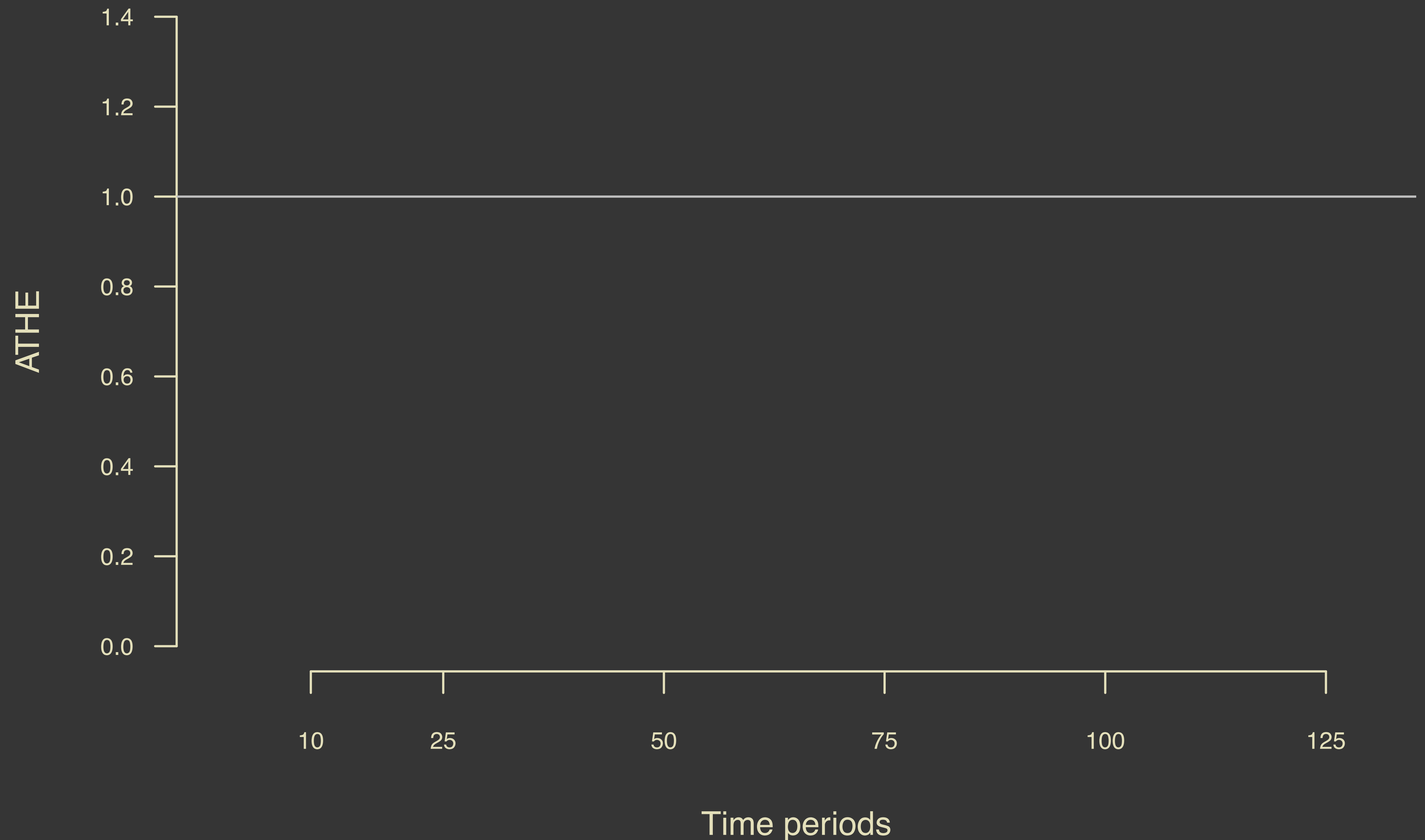


Treatment History Effect: (1,1) vs (0,0)



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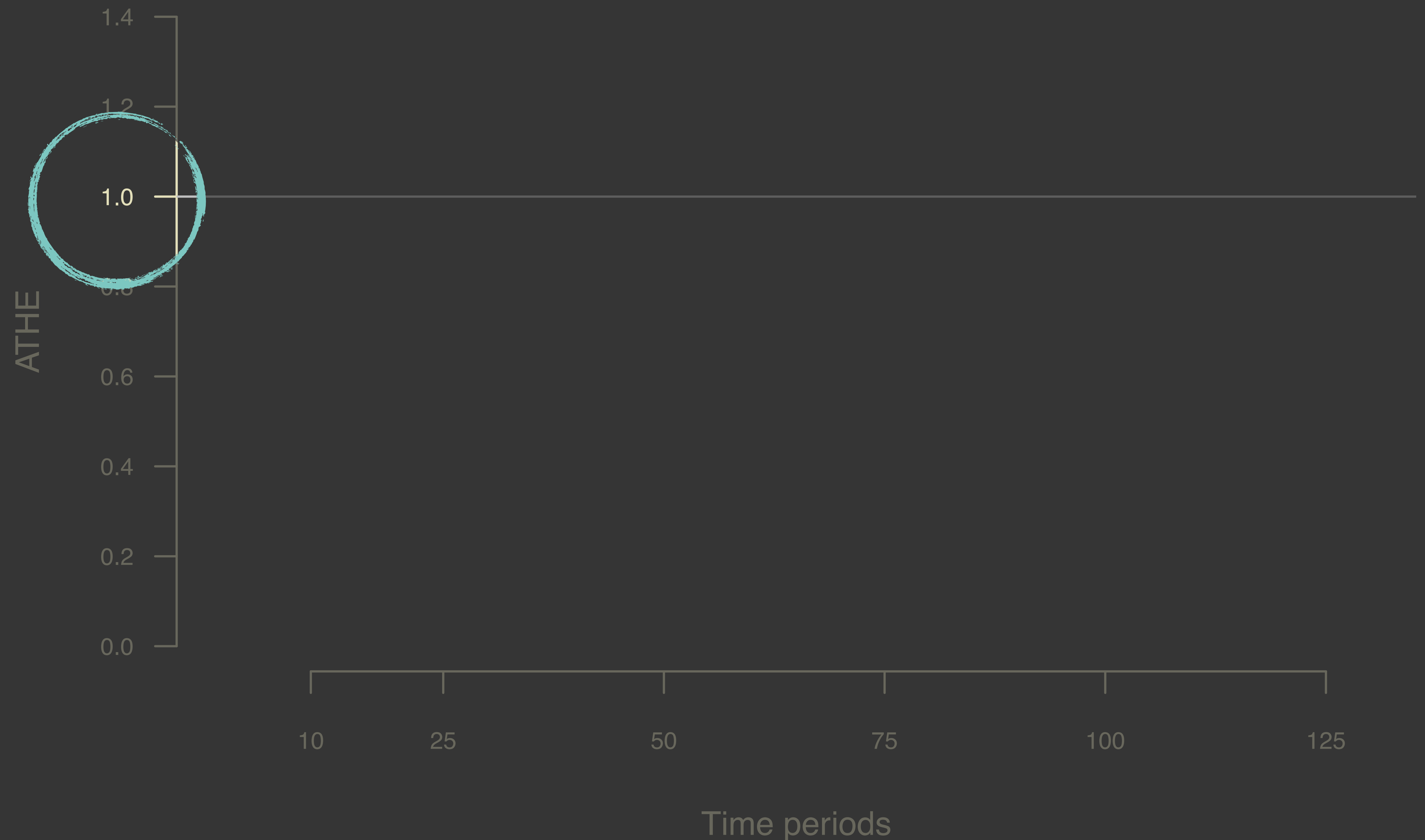

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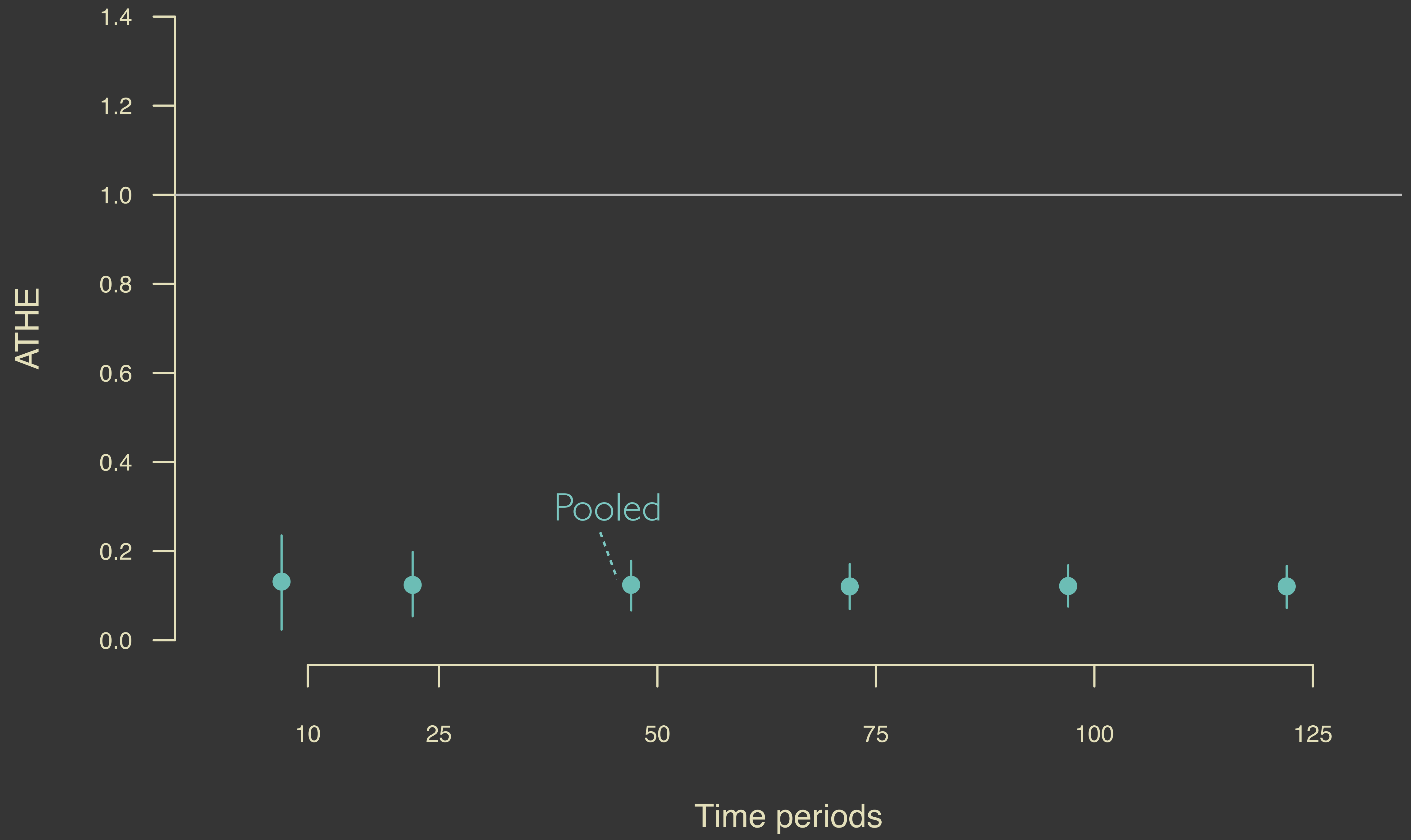

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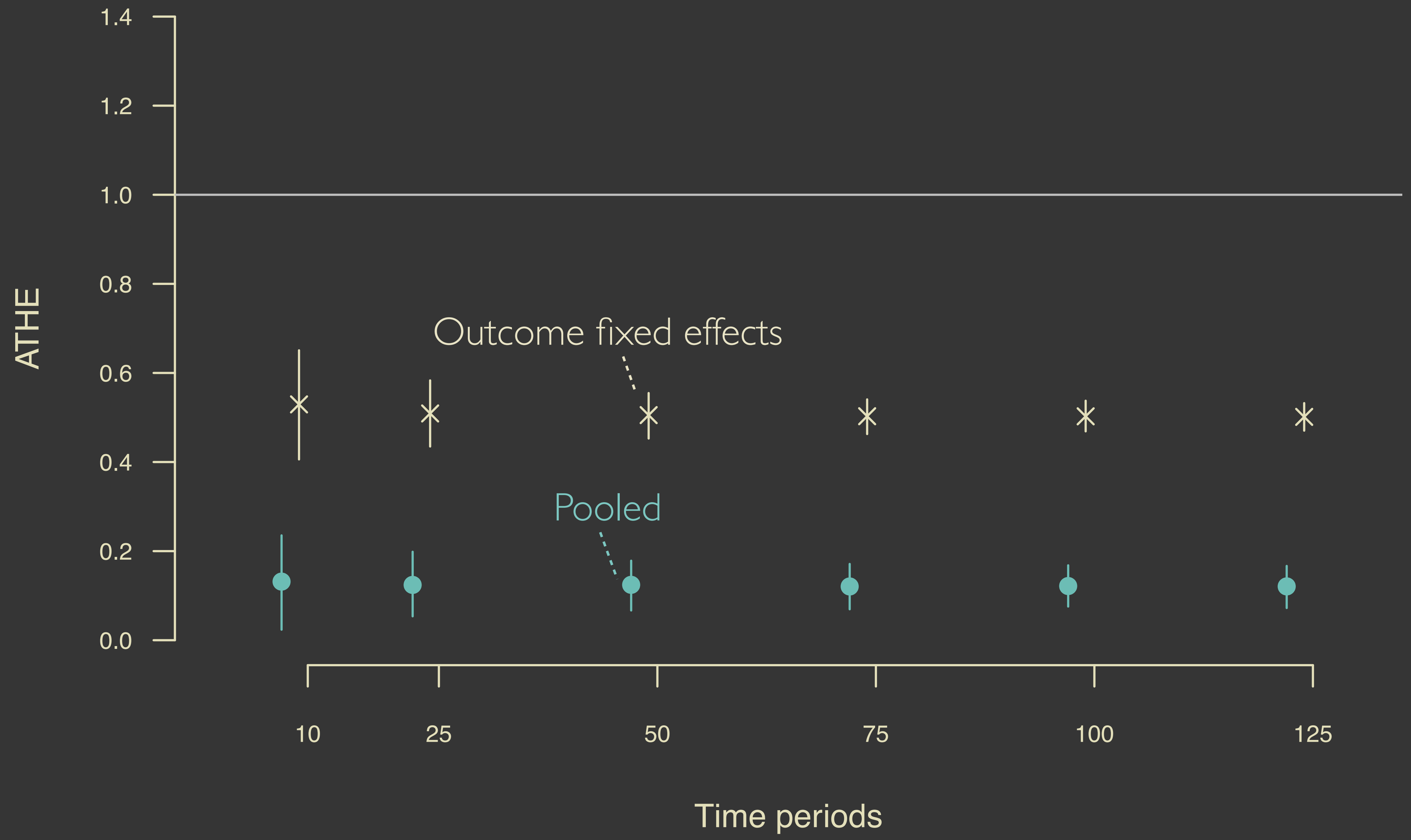

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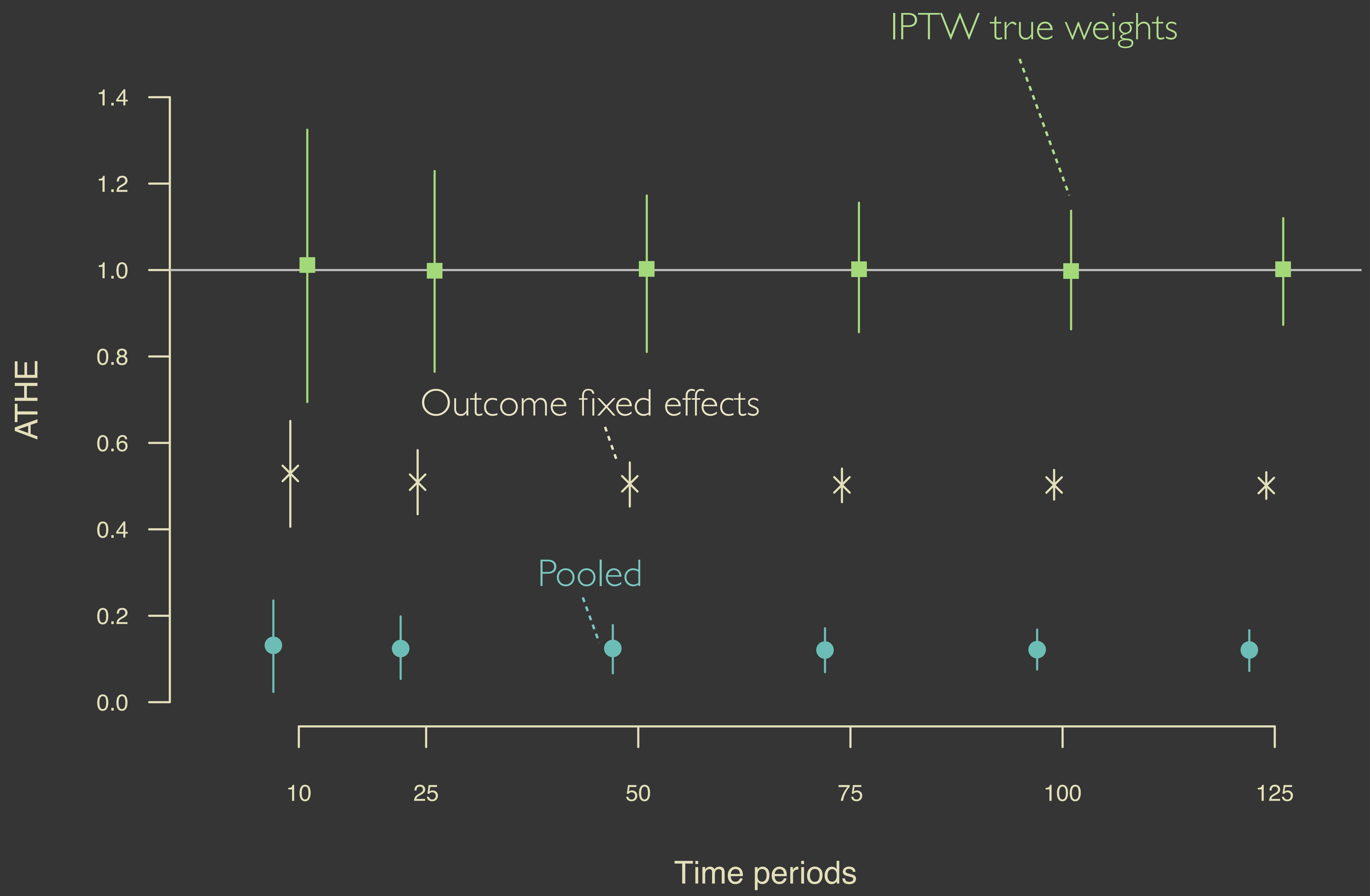

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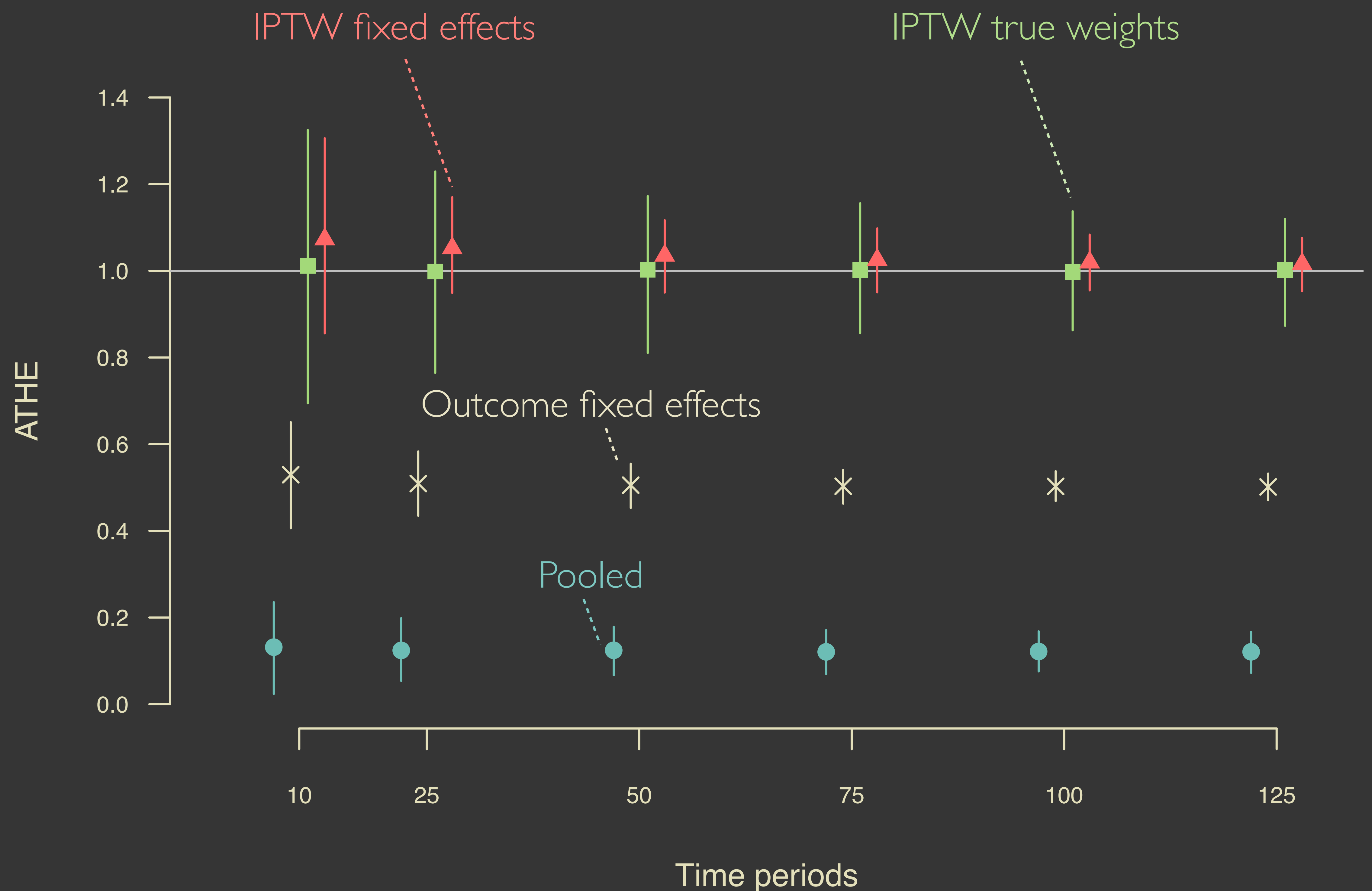

How to make causal inferences with TSCS data

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Very carefully

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Using weights

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Using weights

A fixed effects weighting approach can recover ATHEs and CETs even with unmeasured confounding.