

# Telescope Matching: A Flexible Approach to Estimating Direct Effects

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October 12, 2018

direct effect

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effect of treatment not due to a particular downstream cause

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effect of treatment not due to a particular downstream cause

why do we  
care?

direct effect

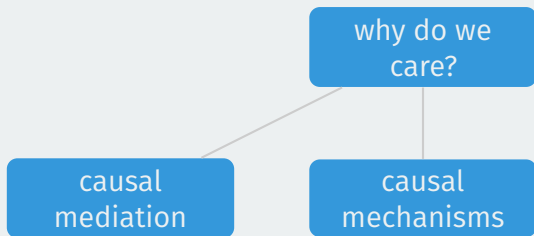
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why do we care?

causal mediation

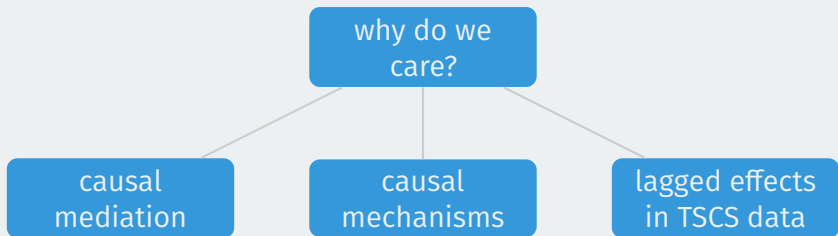
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regression  
& matching



regression  
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posttreatment  
bias

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- robust to (some) model misspecification

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Telescope  
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- consistent for direct effects
- avoids post-treatment bias
- robust to (some) model misspecification
- carries over logic from standard matching



# Roadmap

1. The difficulty of direct effects
2. Our approach: telescope matching
3. Simulating misspecification
4. Application
5. Conclusion

# 1/ The difficulty of direct effects

# Notation

Setting

Effect of frame on immigration media accounts

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$A_i$

Binary treatment  $\in$  {negative frame, positive frame}

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Outcome (support for immigration)

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$Y_i$

Outcome (support for immigration)

$Y_i(a, m)$

Potential outcome

# The Quantity of Interest

Definition (Average Controlled Direct Effect)

$$\tau(m) = E[Y_i(1, m) - Y_i(0, m)]$$



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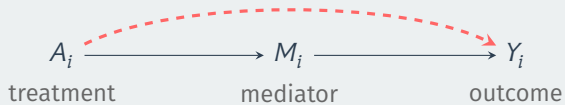
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- Average effect of manipulating  $A_i$  while fixing  $M_i$  to level  $m$
- Easily identified if  $A_i$  and  $M_i$  are randomized but...
- Lots of studies are observational in  $M_i$  or both.

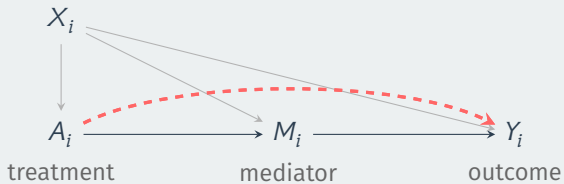
# Confounders

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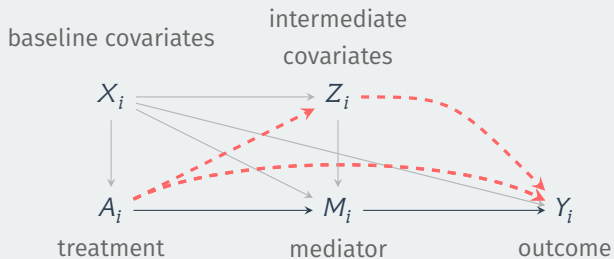


# Confounders

baseline covariates



# Confounders



# Assumptions



## Assumption (Sequential Ignorability)

$$\{Y_i(a, m), M_i(a), Z_i(a)\} \perp\!\!\!\perp A_i | X_i = x$$

$$Y_i(a, m) \perp\!\!\!\perp M_i | A_i = a, X_i = x, Z_i = z$$

*No omitted variables for  $A_i$  given  $X_i$ .*

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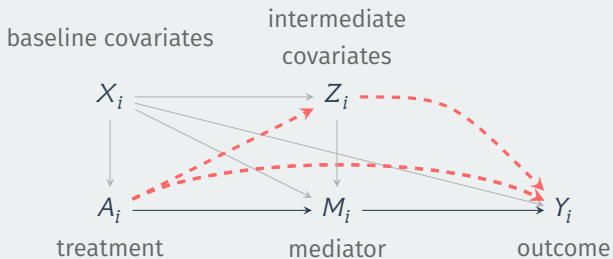
## Assumption (Positivity)

$$0 < P(A_i = 1 | X_i = x) < 1$$

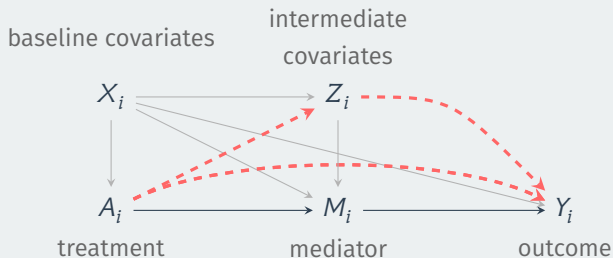
$$0 < P(M_i = 1 | X_i = x, Z_i = z, A_i = a) < 1$$

*Overlap in the covariate distributions across levels of  $A_i$  and  $M_i$*

# The Problem

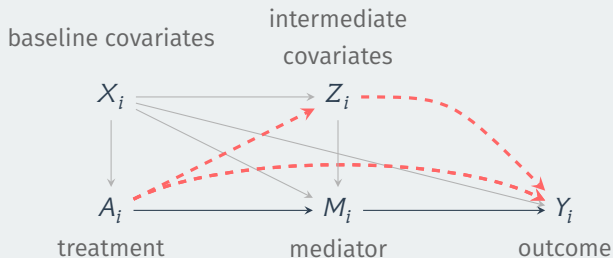


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naive regression/matching of  $Y_i$  on  $X_i$ ,  $A_i$ ,  $M_i$ , and...

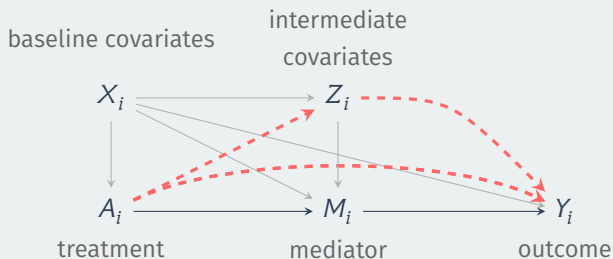
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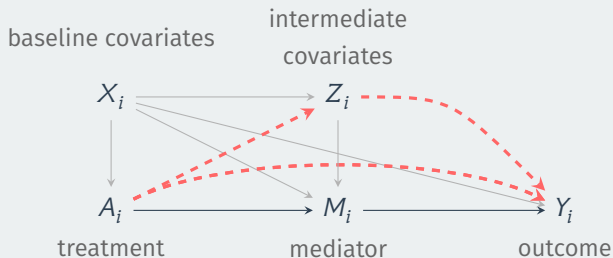


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omitted variable bias  
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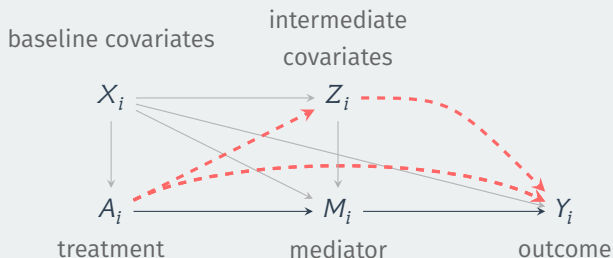
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omitted variable bias  
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post-treatment bias  
for  $A_i$



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Structural  
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Need the correct model for  $\mathbb{E}[Y_i|X_i, A_i, Z_i, M_i]$   
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Inverse  
probability of  
treatment  
weighting  
(IPTW)

Need the correct model for  $\mathbb{P}[M_i|X_i, A_i, Z_i]$   
and  $\mathbb{P}[A_i|X_i]$

## **2/** Our approach: telescope matching

## Telescope matching

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Two-stage matching procedure

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Match  $M_i$   
on  $Z_i$ ,  $A_i$ , and  $X_i$

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Match  $M_i$   
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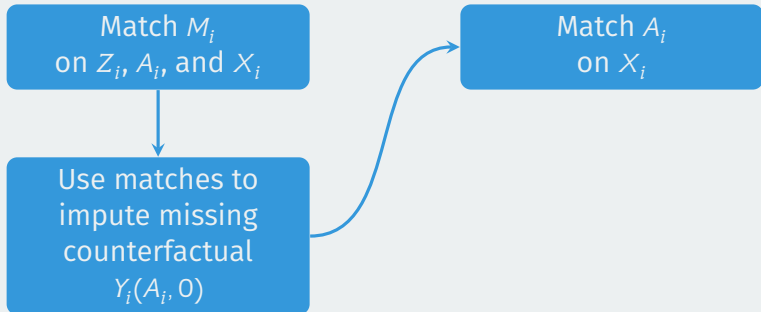


Use matches to  
impute missing  
counterfactual  
 $Y_i(A_i, 0)$



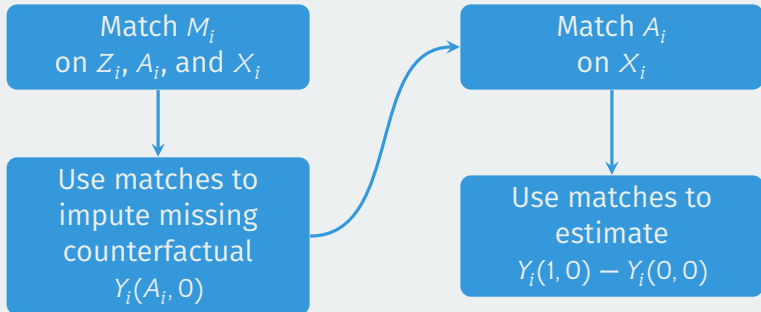
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## Telescope matching

### Two-stage matching procedure



# An imputation problem

Unit	Observed				Potential Outcomes			
	$A_i$	$M_i$	$X_i$	$Z_i$	$Y_i(1,1)$	$Y_i(1,0)$	$Y_i(0,1)$	$Y_i(0,0)$

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5	0	0	9	2	?	?	?	$Y_5$
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$$\tau(0) = E[Y_i(1,0) - Y_i(0,0)]$$

# First stage



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$$A_i = 0$$

$$M_i = 1$$



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# First stage

1. Subset to a particular level of  $A_i$

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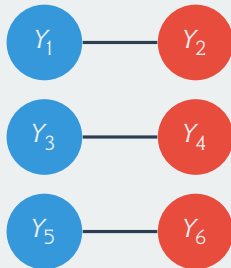
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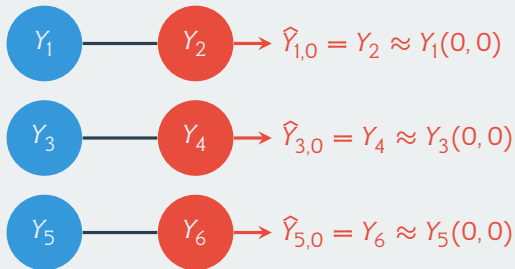
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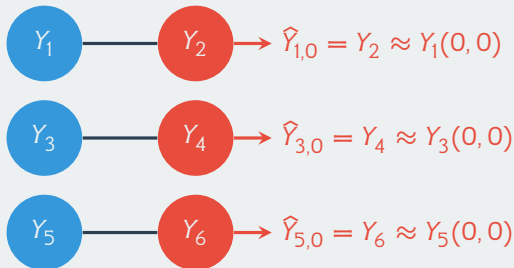
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$$\hat{Y}_{i0} = \begin{cases} Y_i & \text{if } M_i = 0 \\ Y_\ell & \text{if } M_i = 1, M_\ell = 0 \text{ and } \ell \text{ is matched to } i \end{cases}$$

# 1:1 matching example

Unit	Observed				Potential Outcomes			
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3. Take difference in means to estimate ACDE

$$\hat{\tau} = \frac{1}{N} \sum_{i=1}^N \hat{Y}_i(1,0) - \hat{Y}_i(0,0)$$

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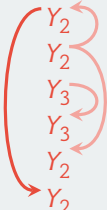
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1	1	1	10	3	$Y_1$	$Y_2$	?	$Y_6$
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$$\hat{\tau} = \frac{1}{6} [(Y_2 - Y_6) + (Y_2 - Y_5) + (Y_3 - Y_5) + (Y_3 - Y_5) + (Y_2 - Y_5) + (Y_2 - Y_6)]$$

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- more robust to model misspecification

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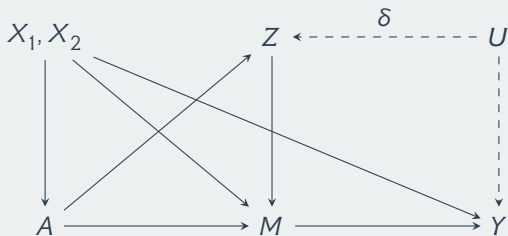
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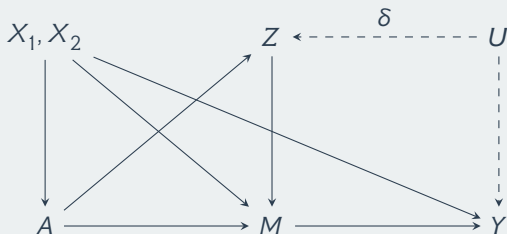
- we follow Otsu and Rai (2017) and resample each contribution to the estimator

## **3/** Simulating misspecification

# Simulation set-up

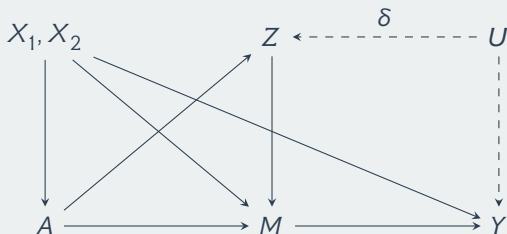


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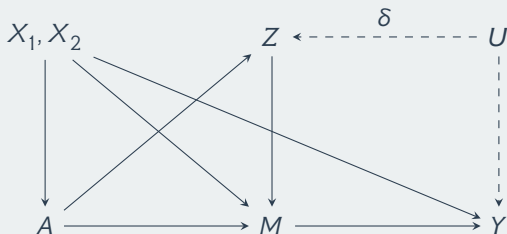
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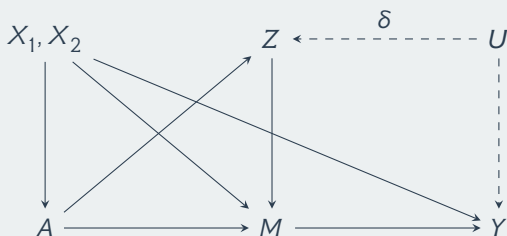
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  - ▶ when  $\delta \neq 0$ , controlling for  $Z$  in a naive regression will induce post-treatment bias.

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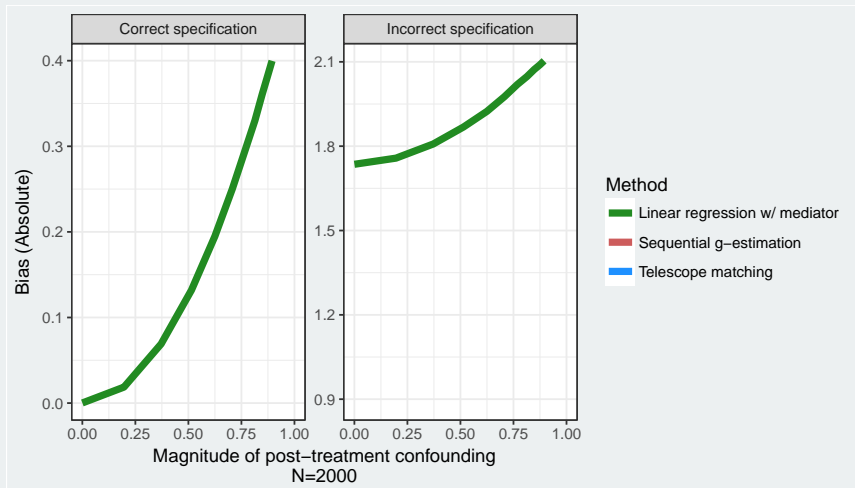
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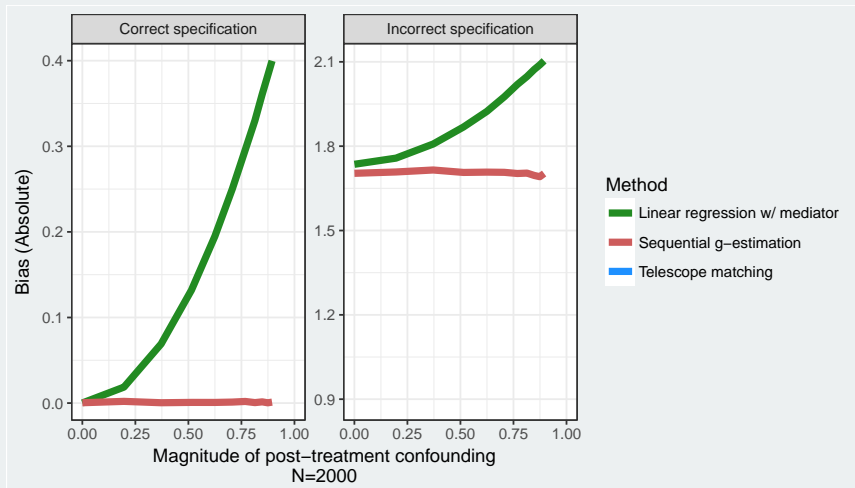
Number of matches per stage = 3

# Simulation results: Bias

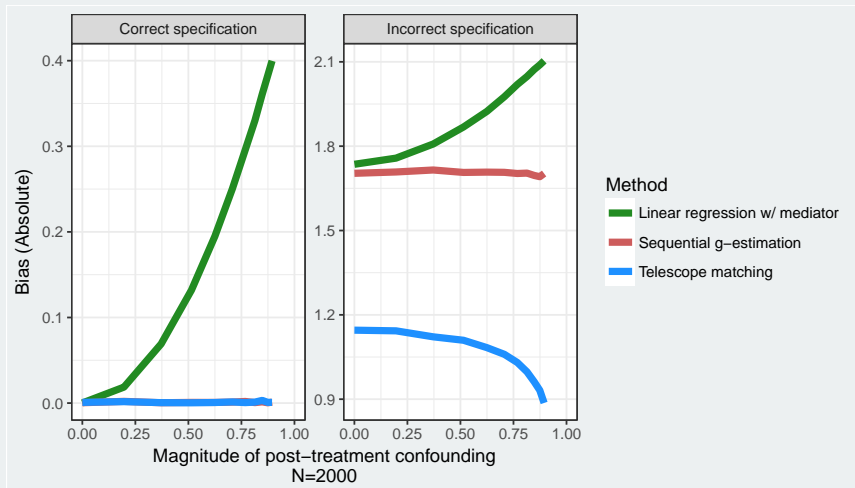
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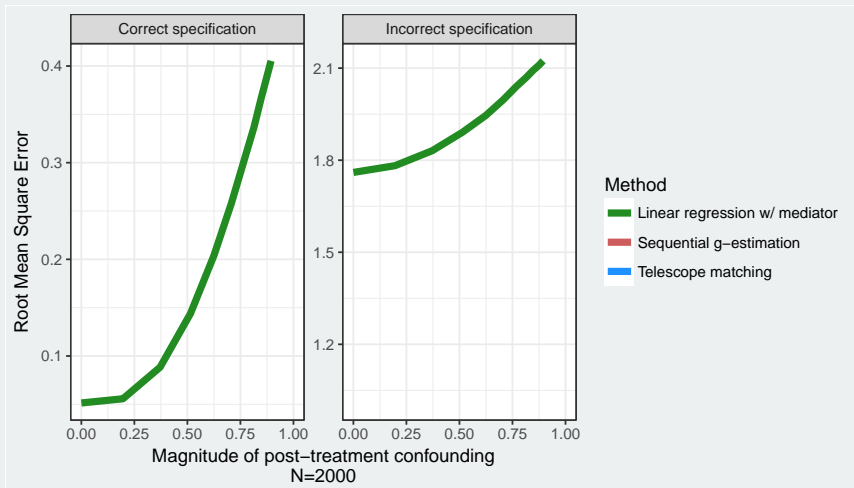
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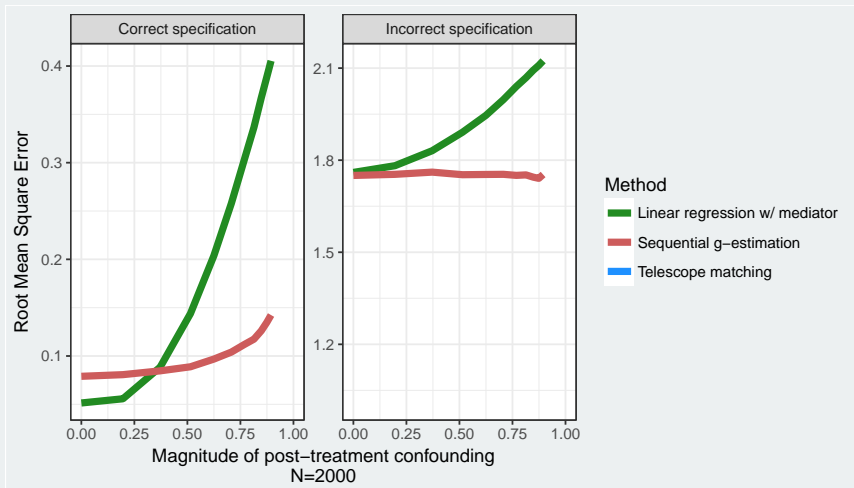
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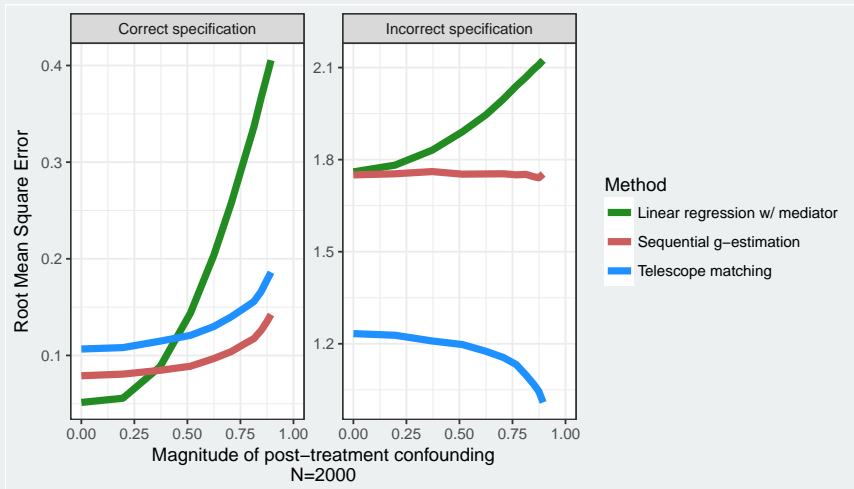
# Simulation results: Root Mean Square Error



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# 4/ Application

- Experiment on effect of media messages on support for immigration.

## Brader, Suhay, Valentino (2008)

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- Main effect: Story w/ negative tone + non-white immigrant reduced support for immigration.

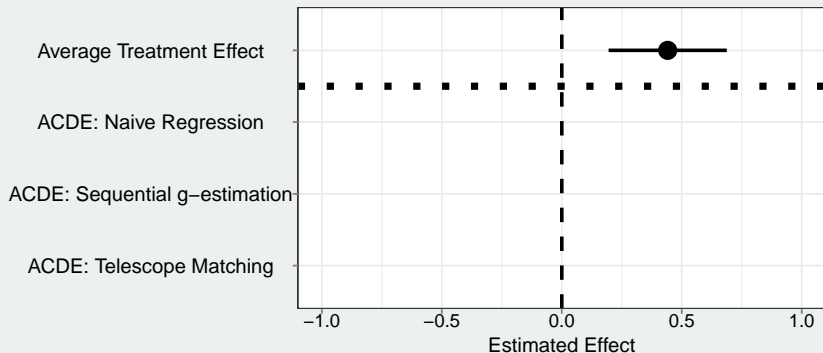
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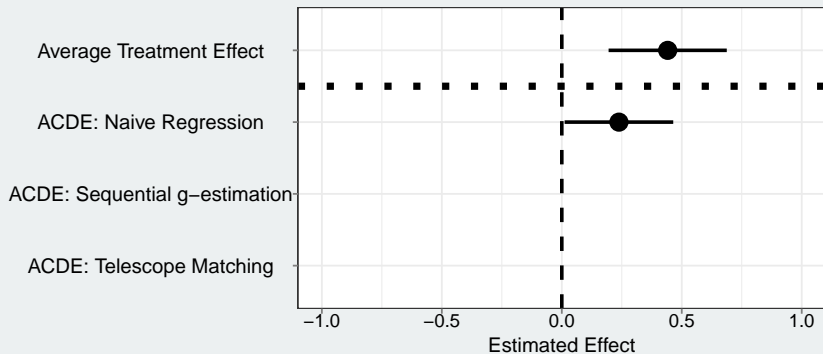
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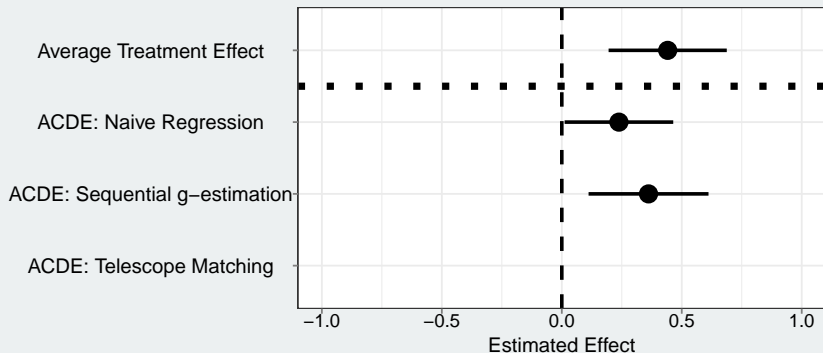




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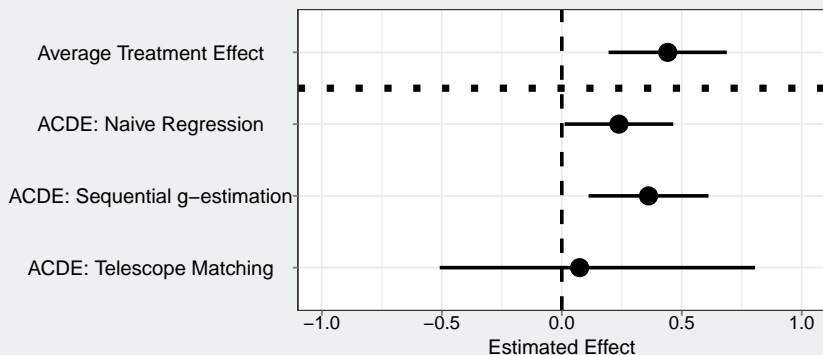


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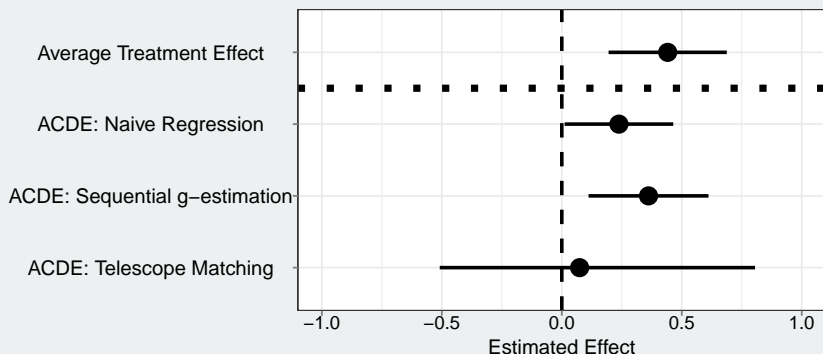


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- $\rightsquigarrow$  Fixing the mediator eliminates most of the treatment effect.

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- How to handle dropping units in the first stage since it induces post-treatment bias?

# Thanks!

- For more information, see:
- [http://www.mattblackwell.org/files/papers/telescope\\_matching](http://www.mattblackwell.org/files/papers/telescope_matching)
- <http://www.mattblackwell.org>
- <https://www.antonstrezhnev.com/>

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